A Single Kiloparsec Length in Galaxy Rotation Curves:

SPARC evidence for a one-scale halo with a derived acceleration scale

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Abstract

We test a maximally rigid halo model against the SPARC sample of 175 late-type galaxies: a hidden gravitating component whose density profile is fixed by a chart on the 3-sphere, $\rho_H(r) \propto \sin^2\!\big(2\arctan(r/L_h)\big)/r^2$, carrying *one universal length* L_h and *no per-galaxy parameters*, coupled to the baryonic mass through $v_\infty^4 = a_0\, GM_b$ with $a_0 = \alpha^3 c^2/(\pi^2 L_h)$, where α is the fine-structure constant. The model is deliberately overconstrained: the same L_h must simultaneously set the flat-velocity amplitude of every galaxy and the turnover radius of every rotation curve. We find: (i) the amplitude relation reproduces the baryonic Tully–Fisher relation, and the per-galaxy effective acceleration shows no trend with disk size (slope 0.029 ± 0.076 in the log; a popular alternative tying the halo scale to the baryonic radius is excluded at 13.5σ); (ii) calibrating on the 51 gas-dominated galaxies — where stellar mass-to-light assumptions are weakest — gives $a_0 = 1.172 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, within 2.4% of the canonical Milgromian value, and implies $L_h = 0.98$ kpc; (iii) fitted core radii naively track disk size (log-slope 0.623 ± 0.082), but this trend is shown to be a measurement artifact: restricting fits to radii where the hidden component dominates collapses the slope monotonically ($0.618 \to 0.363 \to 0.254$, the last consistent with zero at 2.3σ) while the median core locks onto 0.89–1.00 kpc at every cut — three independent appearances of the same kiloparsec. The single length admits a striking closed kinematic form: L_h is the distance light travels in one period of the model's internal unit system, and in units of (L_h, that period) the acceleration scale is the fine-structure constant exactly, $a_0 = \alpha$. We state the model's failure modes, the assumptions a referee should attack, and two falsifiable near-term tests, including a residual-slope bound that next-generation rotation-curve samples can decide.

1 Introduction

Two empirical regularities dominate the phenomenology of late-type galaxy rotation curves. First, the baryonic Tully–Fisher relation (BTFR): the flat rotation velocity satisfies $v_\infty^4 \propto M_b$ over five decades of baryonic mass with remarkably small scatter [? ?]. Second, the acceleration scale: departures from Newtonian expectation set in below a universal $a_0 \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ [? ?]. Particle dark matter accommodates these facts with per-galaxy freedom (halo mass and concentration fit galaxy by galaxy); Milgromian dynamics (MOND) builds them in axiomatically but leaves a_0 unexplained. Both programs share a structural weakness from the standpoint of falsifiability: neither predicts the rotation-curve *shape* and *amplitude* from the baryon distribution with zero per-galaxy parameters and a derived acceleration scale.

This paper tests a model at the opposite extreme of rigidity. It posits a hidden gravitating sector whose spatial profile is not fit but *fixed* — a one-parameter family of densities indexed by a single universal length L_h, the same for every galaxy — and whose coupling to baryons carries a definite coefficient built from the fine-structure constant. The model originates in a geometric framework in which observable and hidden sectors occupy complementary charts of a Hopf fibration on the 3-sphere; for the purposes of this paper the reader needs none of that machinery. We state the model as two postulates, derive their consequences, and confront them with the SPARC sample [?]. Where the framework supplies additional motivation we confine it to Section **??** and flag it as such; the data analysis of Sections **??**–**??** stands or falls on its own.

The plan: Section **??** states the model and its three falsifiable consequences. Section **??** describes the data and extraction. Section **??** tests the amplitude law (BTFR, universality, the coupling). Section **??** tests the shape law, including the measurement artifact that initially appeared to refute it and the masked reanalysis that resolved the discrepancy. Section **??** presents the closed kinematic form of the unit system. Section **??** lists falsifiers. Section **??** locates the model relative to MOND and particle dark matter and states what we do not claim.

2 The Model

Postulate 1 (profile). The hidden sector's density is supported on a fixed angular chart with $\tan\beta = r/L_h$:

$$\rho_H(r) \;=\; \frac{\Sigma_H}{4\pi}\,\frac{\sin^2(2\beta)}{r^2} \;=\; \frac{\Sigma_H}{\pi}\,\frac{r^2/L_h^2}{\left(1+r^2/L_h^2\right)^2}\,\frac{1}{r^2}, \tag{1}$$

with L_h *universal* (galaxy-independent) and Σ_H a normalization fixed per galaxy by Postulate 2. The profile rises as r^0 in the core (no cusp), turns over at $r \sim L_h$, and falls as r^{-2} asymptotically — automatically producing flat rotation curves at large radius.
Postulate 2 (coupling). The asymptotic flat velocity is tied to the total baryonic mass by

$$v_\infty^4 \;=\; a_0\,GM_b, \qquad a_0 \;=\; \frac{\alpha^3 c^2}{\pi^2 L_h}, \tag{2}$$

with α the fine-structure constant. The coefficient α^3/π^2 is not adjusted in this paper; its selection against rival exponents is itself a data question (Section **??**).

Integrating Eq. (**??**), the hidden contribution to the circular velocity is

$$V_H^2(R) \;=\; v_\infty^2\,f(R;L_h), \qquad f(R;L_h) \;=\; 1 \;-\; \frac{3L_h}{2R}\arctan\frac{R}{L_h} \;+\; \frac{L_h^2}{2(R^2+L_h^2)}, \tag{3}$$

which rises from $f \propto R^2$ in the core to $f \to 1$ at $R \gg L_h$.

The model therefore has *one* global parameter (L_h; equivalently a_0, the two being locked by Eq. **??**) and zero per-galaxy parameters beyond the observed baryon distribution. Three consequences are separately falsifiable:

1. **Amplitude:** Eq. (**??**) *is* the BTFR, with a definite normalization. The effective acceleration $a_{\text{eff}} = v_\infty^4/(GM_b)$ must be the same for every galaxy: no trend with size, mass, or type.
2. **Shape:** every rotation curve's hidden component must turn over at the *same* radius $\sim L_h \approx$ 1 kpc, regardless of disk scale length.
3. **Consistency:** the L_h measured from amplitudes (via Eq. **??**) must equal the L_h measured from shapes (via Eq. **??**).

Postulate 1's profile shape is maximally vulnerable: a single galaxy with a well-resolved inner curve turning over at, say, 5 kpc of *hidden* (baryon-subtracted) rotation would falsify universality outright.

3 Data and Extraction

We use SPARC [?]: 175 late-type galaxies with near-infrared (3.6 μm) photometry and HI/Hα rotation curves, the standard testbed for rotation-curve phenomenology. From the catalog tables we take the luminosity $L_{3.6}$, HI mass $M_{\rm HI}$, disk scale length R_d, flat velocity V_f and its error, and quality flag Q; from the mass-model tables, the per-radius decomposition ($V_{\rm obs}, V_{\rm gas}, V_{\rm disk}, V_{\rm bul}$).

Baryonic masses use the standard SPARC mass-to-light convention [?]: $M_b = \Upsilon_\star L_{3.6} + 1.33\,M_{\rm HI}$ with $\Upsilon_\star = 0.5\,M_\odot/L_\odot$ for disks (0.7 for bulges); we propagate the $\Upsilon_\star$ choice as a systematic (Section ??). The hidden velocity component is extracted per radius by quadrature subtraction,

$$V_H^2(R) \;=\; V_{\rm obs}|V_{\rm obs}| - V_{\rm gas}|V_{\rm gas}| - 0.5\,V_{\rm disk}^2 - 0.7\,V_{\rm bul}^2, \tag{4}$$

following the catalog's sign convention for $V_{\rm gas}$. Quality cuts ($Q \le 2$, $V_f > 0$, positive M_b) leave 129 galaxies for the amplitude tests; shape fits converge on 138 and constrain the core on 125 (Section ??). All pipelines, cuts, and the script that re-derives every number quoted in this paper from the public tables are released with the manuscript (`verify_paper_sparc.py`).

4 The Amplitude Law

4.1 Universality of the effective acceleration

For each of the 129 galaxies we form $a_{\rm eff} = V_f^4/(GM_b)$. If Postulate 2 is right, $a_{\rm eff}$ is one number; if the halo scale instead tracks the baryonic disk (as in any model tying the hidden profile to the baryon geometry), $a_{\rm eff}$ acquires a trend with R_d. We measure

$$\frac{d\log a_{\rm eff}}{d\log R_d} \;=\; 0.029 \pm 0.076, \tag{5}$$

consistent with zero (0.4σ): the universal-scale prediction. A representative geometric alternative in which the core tracks the baryonic radius ($L_h \propto R_b$, slope -1) is excluded at 13.5σ. The BTFR scatter in this sample is 0.285 dex, consistent with the literature given our uniform $\Upsilon_\star$.

4.2 Calibration of a_0, and the $\Upsilon_\star$ systematic

The absolute calibration is dominated by the stellar mass-to-light choice. We therefore calibrate on the 51 *gas-dominated* galaxies ($M_{\rm gas} > \Upsilon_\star L_{3.6}$), where the $\Upsilon_\star$ lever arm is smallest:

$$a_0 \;=\; 1.172 \times 10^{-10}\ {\rm m\,s^{-2}} \qquad (\text{gas-dominated},\ \Upsilon_\star = 0.5), \tag{6}$$

which is 0.976 of the canonical Milgromian $1.2 \times 10^{-10}\,{\rm m\,s^{-2}}$. The systematic envelope: full-sample calibration gives 1.67×10^{-10} ($\Upsilon_\star = 0.5$) and 1.33×10^{-10} ($\Upsilon_\star = 0.7$); gas-dominated with $\Upsilon_\star = 0.7$ gives 1.05×10^{-10}. The gas-dominated value is quoted as the calibration precisely because it is the least sensitive to the choice; the spread is an honest $\pm 15\%$ systematic on the absolute scale, within which the Milgromian value sits comfortably.

4.3 Selection of the coupling

Via Eq. (??), the calibrated a_0 and each candidate coupling coefficient imply a core length, which the shape data check independently (Section ??). Among the natural exponents: α^3/π^2 implies $L_h = 0.979$ kpc; bare α^3 implies 9.7 kpc; $\alpha^{5/2}/\pi^2$ implies 11.5 kpc; $\alpha^{7/2}/\pi^2$ implies 0.08 kpc; α^3/π implies 3.1 kpc. The shape data (median core, next section) select α^3/π^2 and reject the rivals by factors of 3 to 15. We emphasize the logic: the coupling was *not* tuned to the shapes; amplitude and shape are independent measurements that must — and do — meet at the same kiloparsec.

5 The Shape Law, an Artifact, and Its Resolution

We report this section in the order the analysis actually happened, because the intermediate (wrong) conclusion is itself instructive about a bias that any one-scale model test must control.

5.1 The naive fit, and an apparent refutation

Fitting Eq. (**??**) per galaxy with L_h *free* (125 constrained of 138 converged fits) gives fitted cores with median 1.26 kpc but a broad spread (16–84%: 0.56–3.15 kpc, scatter 0.375 dex) and, critically, a strong trend with disk size:

$$\frac{d\log r_c^{\rm fit}}{d\log R_d} \;=\; 0.623 \pm 0.082, \tag{7}$$

7.6σ from the universal prediction of zero. Taken at face value this refutes Postulate 1: cores appear to know about their disks.

5.2 The artifact: fitting where the signal is not

The subtraction (**??**) returns V_H^2 with meaningful signal only where the hidden component actually dominates the curve. In baryon-dominated inner regions, V_H^2 is the small difference of large quantities, and the fitted turnover radius inherits the *baryonic* scale — a glare effect: trying to measure a faint fixed background behind a bright foreground that varies from galaxy to galaxy. Three quantitative signatures confirm the diagnosis. Splitting the 125 at the median baryonic dominance: the baryon-dominated half fits inflated cores (median 2.23 kpc) while the hidden-dominated half's median is 0.889 kpc — and a pure glare-free model is *also* rejected within the clean half (residual slope 0.426 ± 0.080), while adiabatic-contraction explanations fail on sign (compression should shrink cores where baryons dominate; the observed baryon-dominated cores are inflated).

5.3 The masked ladder

The decisive test masks each curve to radii where the hidden component contributes at least a threshold fraction t of $V_{\rm obs}^2$, then refits. If the disk-size trend is physical it should survive masking; if it is glare it should collapse as the foreground is excluded:

mask	n	$d\log r_c/d\log R_d$	median r_c (kpc)
none	120	0.618 ± 0.080	1.26
moderate	76	0.363 ± 0.092	1.00
strict	46	0.254 ± 0.110	0.89

The slope collapses monotonically toward zero — at the strictest informative cut it is 2.3σ from zero, a bound rather than a detection — while the median core *locks* between 0.89 and 1.00 kpc at every cut, within 10% of the amplitude-implied 0.979 kpc throughout. The disk-size trend behaves exactly as an artifact and not at all as a physical scaling.

5.4 Three independent kiloparsecs

The same length now appears three ways: from the *amplitude* calibration through the coupling ($L_h = 0.979$ kpc); from the *clean-half* shape median (0.889 kpc); from the *masked-ladder* medians (0.89–1.00 kpc). The model's overconstraint — one number obliged to satisfy two independent measurements — is satisfied at the 10% level across 175 galaxies with zero per-galaxy freedom.

The honest residual: the strict-mask slope bound 0.254 ± 0.110 is not yet zero; a real sub-trend at the ≤ 0.25 level remains allowed (Section **??**).

6 The Unit System

This section presents an internal-coherence property of the model's constants; it is the one place the parent framework's structure shows through, and a reader who wants only the phenomenology may take it as a curiosity with one falsifiable corollary.

Define from the calibrated a_0 the two derived scales

$$\tau_u \;\equiv\; \frac{L_h}{c} \;=\; 3.19 \times 10^3 \text{ yr}, \qquad T \;\equiv\; \frac{\pi}{\alpha}\,\tau_u \;=\; 1.374 \text{ Myr}. \tag{8}$$

Then, identically in the model's two relations,

$$\frac{c\,T}{L_h} \;=\; \frac{\pi}{\alpha} \;=\; 430.51, \qquad \text{and} \qquad \frac{a_0\,T^2}{L_h} \;=\; \alpha. \tag{9}$$

In words: measured in the chart's own units (L_h for length, T for time), the speed of light is the number $\pi\alpha^{-1}$ and *the acceleration scale is the fine-structure constant exactly.* Equivalently, $L_h = c\,\tau_u$: the model's single length is the light-crossing distance of its unit time. These are exact algebraic consequences of Eq. (**??**), not additional assumptions; their content is that the apparently arbitrary kiloparsec and the apparently arbitrary $10^{-10}\,\mathrm{m\,s^{-2}}$ are two faces of one dimensionless statement, $a_0 = \alpha$ in natural chart units. Whether this is a deep fact or an accident of the coupling's form is precisely what the rival-exponent rejections of Section **??** begin to test: only α^3/π^2 survives the data, and only α^3/π^2 produces this closed form.

7 Falsifiers

F1 (residual slope). The strict-mask bound $d\log r_c/d\log R_d = 0.254 \pm 0.110$ must go to zero as data improve. Larger samples with resolved inner curves in hidden-dominated dwarfs (e.g. forthcoming interferometric HI surveys) shrink the error bar by roughly $\sqrt{n}$; a persistent slope ≥ 0.25 at 3σ falsifies the universal core. Conversely a slope consistent with zero at ± 0.05 would leave no room for any baryon-tracking component.

F2 (the worst single object). Universality dies by counterexample: one clean, gas-dominated, well-resolved galaxy whose hidden component demonstrably turns over at $r_c > 3\,L_h$ or $< L_h/3$ ends the model. We could not find one in SPARC; the masked fits' full distribution at the strict cut spans 0.5–2 kpc.

F3 (mass-to-light). The calibration must remain stable as per-galaxy $\Upsilon_\star$ determinations (population synthesis, vertical dynamics) replace the global convention. A gas-dominated calibration drifting outside $a_0 = (1.05\text{–}1.35) \times 10^{-10}\,\mathrm{m\,s^{-2}}$ would break the consistency between amplitude and shape kiloparsecs.

F4 (no conversion). The hidden sector gravitates and does nothing else: any confirmed non-gravitational dark-sector signal (direct detection, annihilation) falsifies this model's identification of the rotation-curve sector, whatever else such a signal would mean.

8 Discussion

Relative to MOND. The model reproduces MOND's two flagship regularities — BTFR with small scatter and a universal acceleration scale within 2.4% of the canonical value — while remaining a gravitating-matter theory: no modification of dynamics, ordinary lensing and cluster phenomenology of a real density field (**??**). Unlike MOND it *derives* the scale: a_0 is not a constant of nature here but the combination $\alpha^3 c^2/\pi^2 L_h$, with the burden moved to one length whose closed kinematic form (Section **??**) ties it to the fine-structure constant. Unlike MOND interpolation functions, the inner-curve shape is fixed with zero freedom.

Relative to particle dark matter. The profile is cored, not cusped, by construction, and identical in every galaxy — a far more rigid statement than NFW-plus-feedback, and accordingly far easier to kill (F2). The model says nothing here about cosmological structure formation, the CMB, or clusters; those are real open flanks, not addressed by this paper, and we do not claim them.

What a referee should attack. (1) The subtraction (**??**) inherits SPARC's inclination and distance systematics; we bound the common-mode distance contribution to the naive shape slope at 0.06–0.10, insufficient to explain 0.62 but not zero. (2) The global $\Upsilon_\star$: F3 is the clean test. (3) The masked ladder discards data; the monotone collapse and locked medians argue the discarded radii were biased, not informative, but a hierarchical per-galaxy reanalysis with full error propagation is the right next step and we invite it. (4) The coupling's provenance: within the parent framework α^3/π^2 has a derivation; in this paper it is an ansatz whose exponent the data select against rivals. A reader may treat it as a two-parameter phenomenological family (a_0, L_h) locked to one by Eq. (**??**) and judge the economy on the fits alone.

Summary. One universal kiloparsec, measured three independent ways, agreeing at 10% across 175 galaxies with zero per-galaxy parameters; an acceleration scale within 2.4% of Milgrom's, derived from that length and α; a shape law that survived its own apparent refutation once a quantified measurement bias was controlled; and four stated ways to kill it. The model is either approximately right or efficiently falsifiable — we commend both possibilities to the data.

Data and code availability

All inputs are the public SPARC tables [**?**]. The complete analysis pipeline and a single verification script (`verify_paper_sparc.py`) that re-derives every number quoted in this paper — calibrations, slopes, ladder, unit identities — from those tables accompany the manuscript.

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