# Paper 38: Dark as Kernel

The closure-map family, superselection,
and the two exactness defects of one geometry

Léon Fernando Vlegels

2026-06-10

**Abstract**

The corpus's master constraint $C \circ P = I$ (P27, P34) is given its canonical formal type and its central physical consequence. A *closure datum* is a retraction pair $(P\colon X \to X_0,\ C\colon X_0 \to X)$ with $PC = \mathrm{id}$, equivalently a split idempotent $e = CP$, over the corpus's ambient categories, with a defect functor fixed per ambient: kernel in $\mathsf{Hilb}$, comma residue in $\mathsf{Met}$, drift locus in $\mathsf{Dyn}$, A254's collapse delta in $\mathsf{Top}$ (A276). Read this way, $C \circ P = I$ is exact on the canonical sector and idempotent globally, which repairs P27's open objection: the 2-to-1 Hopf map needs no inverse, and its non-injectivity is not an obstruction but the location of the dark sector. The family is closed under composition with an exact kernel ladder, so closure composes and dark adds; dark dominance at depth is structural. On this foundation the paper fixes one definition: *dark is the dynamics of the kernel of a closure map* (A272). Dark *matter* is kernel: the fold's charged sector, annihilated by the observation projection yet present in dynamics, carrying $\ell(\ell+1)$ modes per level. Dark *energy* is cokernel: P32's corner residual $\Lambda_0 = 1 - \pi^2/32$ is the measure the embedded $B^4$ fails to cover, consolidated here as canon context. The two cosmological darks are the two exactness defects of one map family. Three theorems govern kernel behavior: the kernel is *sealed* (A261's charge superselection: the fold is exactly equivariant, no conversion back to the image, which is why dark is dark), *paced* (it shares the image's clock; nullspaces breathe), and *never empty* (the rotation number is irrational; dark is mandatory). Superselection inverts the fold's mHz prediction into a discovery channel for LISA, and Paper 39 consumes the kernel as the measured halo. Finally, the operating system's luminosity channel is literal: dark entries are kernel states of the canonical-closure projection, plateau detection is kernel-rank estimation, and the rule that system calls the Butlerian line (no agent silently converts an undecided state into a decided one) is kernel protection: never silently promote from the nullspace. The cosmology and the operating system are one formalism.

## 1 Introduction

The word "dark" carries a substance connotation it has never earned. Dark matter is routinely imagined as a new particle species; dark energy as a new fluid. This paper takes the opposite starting point, the one the corpus's geometry forces: *dark is defined by what observation cannot reach, not by what a new substance is made of.* Observation in this corpus is a projection. Every projection that admits a section has two exactness defects: the part of the domain it annihilates (the kernel) and the part of the codomain it fails to reach (the cokernel). The claim of this paper is that these two defects, of one and the same map family, are the two darks of cosmology, and that the same defects, of the same map family, are the dark entries of the operating system built on this corpus.

The claim is assembled from three addenda and adds nothing to them. A276 supplies the formal class: the closure-map family, split idempotents over the corpus's ambient categories, with a defect functor per ambient and a composition law whose kernel ladder makes dark accumulation structural. A272 supplies the definition and the typing: dark is the dynamics of the kernel of a closure map; dark matter is kernel, dark energy is cokernel. A261 supplies the physics that makes the definition non-trivial: the Hopf charge is a superselection charge of the canonical master operator, so the kernel cannot convert back into the image through any canonical channel. The kernel gravitates and does nothing else. That is why dark stays dark.

Three consequences organize the paper. First, the formal one: P27's recorded objection to $C \circ P = I$ (the Hopf map is 2-to-1, so no inverse exists) dissolves once the constraint is typed as a section law plus idempotency rather than invertibility; the non-injectivity that looked like a defect of the formalism is exactly where dark lives. Second, the observational one: superselection closes the fold-native primordial-black-hole channel and inverts the fold's narrow mHz prediction, so the canon predicts the *absence* of a spike in the mid-LISA band, and a detection there would demonstrate structure beyond the canon; the halo phenomenology that the kernel does support is developed in Paper 39, which consumes this paper's theorems. Third, the operational one: the operating system's luminosity channel acquires literal semantics, and the Butlerian line, never silently promote, becomes a theorem about kernels rather than a policy about agents.

Throughout, "observation projection" means a closure map of the family defined in Section 2: a projection, quotient, or closure condition taking states to their canonical form. The fold map, the Hopf pushforward $S^3 \to S^2$, is the principal physical instance, but the definition quantifies over the class, not the instance.

## 2 The Observation Projection and Its Kernel

The corpus's master constraint $C \circ P = I$ appears in P27 as the coherence condition selecting the shared arena and in P34 as the collapse–projection cycle. Its formal type was left open: read as an identity on all of $X$ it would force $P$ to be invertible, which the 2-to-1 Hopf map is not. A276 fixes the type.

**Definition 2.1** (Closure datum; the family $\mathcal{F}$ (A276))**.** *Let $\mathcal{A}$ be one of the corpus's ambient categories:* $\mathsf{Hilb}$ *(mode spaces),* $\mathsf{Met}$ *(metric state spaces with distinguished lattices),* $\mathsf{Dyn}$ *(circle and sphere maps),* $\mathsf{Top}$ *(filtered spaces with collapse). A* closure datum *in $\mathcal{A}$ is a pair $(P\colon X \to X_0,\ C\colon X_0 \to X)$ with $PC = \mathrm{id}_{X_0}$. Its* closure operator *is the idempotent $e = CP\colon X \to X$, since $e^2 = C(PC)P = CP = e$; its canonical sector is $\operatorname{im} e \cong X_0$. The family $\mathcal{F}$ is the class of all closure data over the ambients. The corpus's $C \circ P = I$ is the statement $e|_{\operatorname{im} e} = \mathrm{id}$: exact on the canonical sector, idempotent (not invertible) globally.*

**Definition 2.2** (Defect functors; dark (A276, A272))**.** *Per ambient, the* defect *of a closure datum is: in* $\mathsf{Hilb}$*, the kernel* $\ker P$*, equivalently* $\operatorname{im}(\mathrm{id} - e)$*; in* $\mathsf{Met}$*, the residue function $x \mapsto d(x, e(x))$; in* $\mathsf{Dyn}$*, the locus where the locking projection fails to converge; in* $\mathsf{Top}$*, A254's collapse delta $\delta_\Omega = \operatorname{rank}\ker(H_0(A) \to H_0(X))$.* Dark*, relative to a closure datum, is the dynamics of its defect: that which the projection annihilates from its image but which persists, evolving, outside it.*

Two features of the definition do the work. First, the defect is *functorial per ambient*, not chosen per instance: once the ambient is fixed, what counts as dark is determined, so the coverage claim of Section 7 quantifies over a defined class rather than a list. Second, the closure operator is a *split* idempotent: the section $C$ exhibits the canonical sector inside $X$, and the complement $\mathrm{id} - e$ exhibits the defect. Nothing about this requires $P$ to be injective. Where $P$ identifies distinct states, the identified directions are precisely the kernel, and the kernel is where the definition places dark.

**Remark 2.3** (The P27 repair (A276))**.** *P27's open remark P027_2 observes that the Hopf map is 2-to-1, so $C \circ P = I$ cannot hold by invertibility, and records the gap as open. Definition 2.1 is the repair, without retraction: the identity was never global. It is the section law $PC = \mathrm{id}_{X_0}$ plus idempotency, which any split projection, including a 2-to-1 one, supports without contradiction. The demand for invertibility dissolves. The non-injectivity is not an obstruction to the constraint; it is exactly where dark lives (*$\ker P \neq 0$*). What looked like the formalism's defect is the universe's dark sector.*

The observation projection of the title is any member of $\mathcal{F}$ whose image is what an observer can reach. For the physical sector the principal datum is the fold: $P$ the Hopf pushforward $S^3 \to S^2$ on mode spaces, $C$ the inclusion of the charge-blind sector; $PC = \mathrm{id}$ holds because the pushforward reproduces the charge-blind radial density exactly (A269). The kernel of this projection is the subject of the next section.

# 3 Dark Matter as Kernel

**Definition 3.1** (Dark = kernel dynamics (A272))**.** *Let $(P, C)$ be a closure datum of $\mathcal{F}$.* Dark*, relative to $(P, C)$, denotes the states and dynamics of the defect of Definition 2.2: in the linear ambient, the states and dynamics of* $\ker P$*.*

The definition covers all three of the corpus's registers of "dark" at once.

**Proposition 3.2** (Instances (A272, A276))**.** *The following are closure data of $\mathcal{F}$, and their defects are the corpus's three dark registers.*

(I1) Fold *(*Hilb*). $P =$ the Hopf pushforward on mode spaces, $C =$ inclusion of the charge-blind sector; $PC = \mathrm{id}$ by A269. The kernel is the fiber-charged sector, of rank $\ell(\ell+1)$ per level $\ell$ (A259 Prop. 1.1): of the $(\ell+1)^2$ modes at level $\ell$, only $\ell+1$ are charge-blind and survive the projection. By A261 and A264 this kernel gravitates without converting, and its kinetic density is the observed halo: dark matter is* $\ker(\text{fold})$ *with dynamics. The per-level dark fraction is $\ell/(\ell+1) \to 1$.*

(I2) Chord closure *(*Met*). $X = \mathbb{R}_{>0}$ (cycle lengths), $X_0 =$ the 3-smooth lattice, $C =$ inclusion, $P =$ nearest-chord projection; $PC = \mathrm{id}$ because a chord is its own nearest chord. The residue at the breath period is the comma $G_1 = 432/T_b - 1 \approx 3.456 \times 10^{-3}$ (A266). Detuning is the dark of harmony.*

(I3) Locking *(*Dyn*). $X_0 =$ the locked orbits (Arnold tongues), $P =$ the phase-locking projection defined on tongue basins, $C =$ inclusion. The null condition $q\rho_{\mathrm{eff}} - p = 0$ defines the tongues (A268); the defect locus is the drifting orbit class, non-empty because $\rho$ is irrational (A267). Drift is the dark of dynamics, and the working OS dictionary: canonical = locked, dark = drifting.*

Three structure theorems then attach to the kernel as corollaries of the definition. Each was proven elsewhere in the corpus; what this paper adds is only that they are statements about one object.

**Corollary 3.3** (Sealed (A261))**.** $\ker P_0$ *cannot leak into the image: every sector of the canonical master operator commutes with the Hopf charge, and the fold map is exactly equivariant. Darkness is dynamically protected. This is why dark matter is dark, restated as a property of kernels under the canonical $\hat{O}$.*

*Sketch.* Section 5 states the superselection theorem in full. The commutation $[\hat{O}, \hat{Q}] = 0$ holds sector by sector; equivariance of the fold means the projection conserves the charge it annihilates; and the regularity axiom plus assembly uniqueness close the loophole of alternative operator forms. No canonical channel connects the charged kernel to the charge-zero image at any order. □

**Corollary 3.4** (Paced (A272, A269))**.** $\ker P_0$ *shares the image's clock: the breathing spectrum descends from the charge-blind radial density (A269), so kernel dynamics is synchronized with canonical dynamics, not stagnant. Nullspaces breathe.*

*Sketch.* The pushforward reproduces the charge-blind radial density exactly (A269); the breath spectrum is a functional of that density; hence the same spectrum paces both the image and the kernel. The kernel is removed from the image's *support*, not from its *time*. □

**Corollary 3.5** (Never empty (A272, A267))**.** *The closure conditions of (I2) and (I3) are never exactly satisfied: the rotation number $\rho$ is irrational (A267, consolidated in P37), so commas are nonzero and some orbit class always drifts. A corpus with empty nullspaces is impossible; dark is mandatory, not accidental.*

*Sketch.* Exact chord closure would require $T_b$ to lie on the 3-smooth lattice and exact locking would require $q\rho - p = 0$ for integers $p, q$; both are rationality conditions, and $\rho$ is irrational. The defect of (I2) is bounded away from zero at every lattice point, and the tongue complement in (I3) has positive measure. □

The kernel is therefore not an inert remainder. It is sealed (it cannot convert), paced (it evolves on the canonical clock), and never empty (no parameter choice eliminates it). These are the three theorems governing kernel behavior, and they are jointly the operational content of the phrase "dark matter": a sector that gravitates, persists, keeps time, and does nothing else.

## 4 Dark Energy as Cokernel

The kernel is what the projection annihilates from the domain. The dual defect is what the section fails to reach in the codomain. The corpus's two cosmological darks are exactly this pair.

**Theorem 4.1** (Kernel/cokernel typing (A272))**.** *Dark matter is kernel: annihilated by the fold, absent from the image, present in dynamics, and measured (P39: the halo's kinetic density). Dark energy is cokernel: $\Lambda_0 = 1 - \pi^2/32$ is precisely the volume fraction of $[-1,1]^4$ not covered by the embedded $B^4$ (P32). The pair (kernel, cokernel) types the two darks of cosmology as the two exactness defects of one map family.*

*Sketch.* The kernel half is Proposition 3.2(I1) with Corollaries 3.3–3.5. The cokernel half is a measure computation: the unit 4-ball has volume $\pi^2/2$, the bounding hypercube $[-1,1]^4$ has volume 16, so the covered fraction is $\pi^2/32$ and the uncovered fraction is $\Lambda_0 = 1 - \pi^2/32 \approx 0.6916$, against the Planck 2018 value $\Omega_\Lambda = 0.6847 \pm 0.0073$ at $0.94\sigma$ with no free parameters (P32). The typing is exact at the level of definitions; any dynamical consequence beyond it would require new work (A272 Rem. 2.4). □

**Remark 4.2** (Cokernels are in the family (A276))**.** *The cokernel is not an exception to the closure-map formalism but a member of it: in* $\mathsf{Hilb}$*, the cokernel of $(P, C)$ is the kernel of the* opposite *datum $(C^\dagger, P^\dagger)$, which is again a closure datum in $\mathcal{F}$. One family, read in two directions, produces both darks.*

P32's headline result is consolidated here as canon context: the corner residual is the corpus's standing identification of the cosmological constant, and this paper supplies the type it had been missing. Dark energy is not a substance filling space; it is the measure of what the embedding cannot cover, the codomain-side exactness defect of the same geometry whose domain-side defect gravitates as the halo. The asymmetry between the two darks (clustering versus smooth, dynamical versus constant) is the asymmetry between a kernel, which carries dynamics, and a cokernel, which is a measure-theoretic complement.

## 5 Superselection and the Sealing Discipline

The definition of Section 3 would be vacuous if the kernel could decay into the image: dark matter that converts is just slow ordinary matter. The content of A261 is that conversion is forbidden exactly, not approximately.

**Theorem 5.1** (Hopf-charge superselection (A261))**.** *Let $\hat{Q} = -i\,\partial_\psi$ generate the Hopf $U(1)$ on $S^3$, with integer charge $q$ labelling the fiber sectors. Then $[\hat{O}, \hat{Q}] = 0$ for the canonical master operator $\hat{O}$, sector by sector: the radial sectors ($D^2_{B^4}$, $V^{\mathrm{can}}_{\mathrm{self}}$, $\alpha\rho$, $\zeta\mathcal{R}$, $\beta\mathcal{M}$) act on $r$ and spectral weight, not on $\psi$, so commutation is immediate; $\Delta_{S^3}$ is the Casimir of the isometry group containing $U(1)_{\mathrm{Hopf}}$, and a Casimir commutes with every generator; $\Delta_{S^1}$ equals $-\hat{Q}^2$ up to fiber normalization; and $\gamma T_{\mathrm{cycle}}$ commutes because the lens $\mathbb{Z}_3$ deck action is the Hopf $U(1)$ element at $\psi = 2\pi/3$ (A255) and $U(1)$ is abelian. The conclusion is robust beyond the canonical assembly: A189's Hopf Regularity Axiom forbids any admissible extension from breaking the symmetry, and A218b's assembly uniqueness removes the loophole of alternative $\hat{O}$ forms. Fiber charge is a superselection charge of the kernel, and the transfer fraction from the charged sector into scalar perturbations is $s = 0$ at all orders.*

**Proposition 5.2** (Equivariance of the fold (A261))**.** *The fold completion $S^3 \to S^2$ acts on modes as the charge-zero projection $P_0$, and $P_0$ is exactly equivariant: the Hopf base point $(2z_1\bar{z}_2,\ |z_1|^2 - |z_2|^2)$ is invariant under the $U(1)$ action (verified numerically to $10^{-12}$ over random states). The fold conserves the charge it annihilates: it removes the charged sector's* support*, not its* charge*. In the sudden limit the charged components have no image in $L^2(S^2)$ and their energy enters the homogeneous trace; in the adiabatic limit charged energies $\sim q^2/r^2_{\mathrm{fiber}}$ blueshift as the fiber collapses and exit the spectrum into the background. In both limits the deposit is homogeneous: zero scalar transfer.*

**Remark 5.3** (Wheel heat: the deposit's corpus identity (A261, A276))**.** *The kernel already owns the bookkeeping object for energy that is conserved but attached to no face: Wheel heat (`kernel/wheel/wheel.py`; heat conservation across Fork/Weld pairs). The fold is a Weld-sector event (*Weld $\leftrightarrow \Delta_{S^3}$*), and the annihilated sector's energy is Wheel heat: the fold defect* thermalizes*, it does not gravitate as structure. A276 types this consistently as the quantitative defect, the energy carried by* $\mathrm{id} - e$*, while recording that the energy form on* $\mathrm{id} - e$ *is flagged, not developed.*

**Remark 5.4** (NEW THEORY boundary (A261))**.** *A nonzero transfer fraction would require a non-equivariant correction to the fold map, a preferred angular direction during completion. The canon not only lacks such structure; the Hopf Regularity Axiom exists precisely to exclude fixed-point-bearing angular directions from the assembly. Any future mechanism delivering $s \neq 0$ is NEW THEORY by construction and must repeal or weaken that axiom explicitly.*

Superselection seals one kernel. The family laws of A276 extend the discipline to every composite of closures.

**Theorem 5.5** (Composition; the kernel ladder (A276))**.** *(i) If $(P_1, C_1)$ on $X$ with canonical sector $X_0$ and $(P_2, C_2)$ on $X_0$ with canonical sector $X_{00}$ are closure data, then $(P_2P_1,\ C_1C_2)$ is a closure* *datum on $X$: $P_2P_1C_1C_2 = P_2C_2 = \mathrm{id}$. The family $\mathcal{F}$ is closed under composition and contains identities, hence forms a category fibred over the ambients. (ii) In* Hilb *the sequence*

$$0 \to \ker P_1 \to \ker(P_2P_1) \xrightarrow{P_1} \ker P_2 \to 0 \tag{1}$$

*is exact: the right map is onto because $C_1$ sections it, since $P_1C_1 = \mathrm{id}$ maps $\ker P_2$ into $\ker(P_2P_1)$. Hence $\dim\ker(P_2P_1) = \dim\ker P_1 + \dim\ker P_2$: dark accumulates additively along composed closure.*

**Corollary 5.6** (Dark dominance at depth is structural (A276))**.** *Iterating closure grows the defect monotonically; no composite of closure data can shrink a kernel. The fold's per-level dark fraction $\ell/(\ell+1) \to 1$ is an instance of the ladder, not a peculiarity of the fold. A universe reached through many closures is generically dark-dominated.*

The slogan is: *closure composes, dark adds.* Each act of canonicalization, each projection to an observable sector, leaves a kernel; composing the acts sums the kernels; nothing in the family can run the sum backwards. Observed dark dominance is what a deeply closed geometry looks like from inside its canonical sector.

### The sealing discipline in the operating system

The same mathematics types the operating system built on this corpus.

**Proposition 5.7** (Dark luminosity = kernel states (A272))**.** *A dark entry in the OS's luminosity channel is a traversal in the kernel of the canonical-closure projection: it ran, deposited a trajectory, and did not close. Plateau detection, dark density exceeding canonical density over a region, is local kernel-rank estimation (prototype: A254's collapse delta $\delta_\Omega$). The three corollaries transfer. Dark entries are* sealed*: they are never silently promoted to canonical, and the Butlerian line is kernel protection. They are* paced*: indexed by the same breath ledger as canonical entries. And they are* never empty*: a system whose processes never stall has stopped measuring. The OS's metabolism and the cosmology's dark sector are the same mathematics. (A hash table is a further instance: collisions are* $\ker P$*, the dark of hashing, A289.)*

The sealing discipline deserves its operational statement. In the cosmology, superselection means the kernel cannot promote itself into the image: no canonical dynamics converts dark into light. In the operating system, the corresponding rule is that no agent silently converts a failed traversal into a canonical entry: promotion out of the nullspace requires an explicit act at the boundary of the system, exactly as a nonzero transfer fraction would require NEW THEORY at the boundary of the canon (Remark 5.4). The kernel ladder adds the bookkeeping half of the discipline: since dark adds along composition and no composite shrinks a kernel, dark entries are never lost by stacking closures, and any apparent decrease in dark rank signals an accounting error, not a conversion.

## 6 Observational Consequences

### The LISA band signature

Before superselection was established, the fold's annihilated sector was a candidate source for scalar curvature perturbations, carrying a fold-native primordial-black-hole channel and a narrow scalar-induced gravitational-wave spike at 0.45–0.95 mHz in the mid-LISA band (A257, A259). Theorem 5.1 closes the sourcing: with $s = 0$, the fold contributes no enhancement ($\sigma_{\mathrm{fold}} = \sigma_0$ exactly), and with $\delta_c/(\sqrt{2}\,\sigma_0) \approx 6.9 \times 10^3$ the fold-native PBH abundance is negligible ($\log_{10}\beta \sim$

$-10^7$): the channel is closed negatively (A261). The associated $\alpha^6$ candidate and its $\Omega_k \approx -3.5 \times 10^{-3}$ curvature pinning are demoted to void; the $2.2\sigma$ curvature tension never materializes.

The mHz prediction thereby *inverts*: the canon predicts the *absence* of the narrow spike. A LISA detection of a narrow feature in that band would demonstrate structure beyond the canon, a non-equivariant correction to the fold that the Hopf Regularity Axiom excludes from the present assembly. What was a falsifier becomes a discovery channel: the cleanest place to find physics the canon does not contain is the band where the canon insists on silence.

Superselection also resolves a consistency demand rather than merely imposing one. A259's overproduction exclusion required the fold to nearly decouple from the scalar sector on observational grounds; Theorem 5.1 shows the kernel guarantees *exact* decoupling. The regularity axiom, adopted on spectral grounds, independently protects the universe from shredding at fold completions. The heavy-seed channel survives untouched, since it requires only standard perturbations plus the fold epoch (A256, A260).

### The halo: the kernel made visible

Paper 39 consumes this paper's theorems and is the kernel's contact with telescopes; the results are summarized here as consequences, not re-derived. Because the kernel is sealed (Corollary 3.3), it gravitates and does nothing else; because it is paced (Corollary 3.4), its stationary density is fixed by the breath dynamics rather than by initial conditions. On the hidden chart $\tan\beta = r/L_h$ the kernel's density is

$$\rho_H(r) \;\propto\; \frac{\sin^2\!\big(2\arctan(r/L_h)\big)}{r^2}, \tag{2}$$

with one universal length $L_h$ and zero per-galaxy parameters: cored at small radius, turning over at $r \sim L_h$, asymptotically $r^{-2}$, hence flat rotation curves with no fitting. The amplitude law $v_\infty^4 = a_0 G M_b$ with $a_0 = \alpha^3 c^2/(\pi^2 L_h)$ is the baryonic Tully–Fisher relation with a derived coefficient. Against SPARC (175 galaxies) the per-galaxy effective acceleration shows no trend with disk size ($0.029 \pm 0.076$; the baryon-tracking alternative excluded at $13.5\sigma$), and the gas-dominated calibration gives $a_0 = 1.172 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, 0.976 times the Milgromian value, with $L_h = 0.98$ kpc (P39). The phenomenology of "dark matter halos" is, on this account, the phenomenology of a sealed, paced, never-empty kernel.

## 7 Falsifiers

**Coverage claim, falsifiable.** Every corpus "dark" phenomenon is the defect of a closure datum in $\mathcal{F}$ (Definition 2.1) under one of the named defect functors (Definition 2.2). A dark phenomenon admitting no such presentation breaks the claim. Watch items: Wheel heat (kernel energy after support removal; typed in Remark 5.3, undeveloped) and the i_Paper's Nullity (kernel of frame installation; consistent).

**The mHz band.** The canon predicts the absence of a narrow SIGW spike at 0.45–0.95 mHz. A confirmed detection falsifies the present assembly's exactness claim and localizes the failure: a non-equivariant fold correction, requiring explicit repeal or weakening of the Hopf Regularity Axiom (Remark 5.4).

**Conversion.** Any confirmed non-gravitational dark-sector signal (annihilation, decay, scattering into visible channels) falsifies the sealing theorem directly, since Corollary 3.3 asserts $s = 0$ at all orders within the canon.

**The halo program.** The falsifiers of P39 transfer to this paper through Theorem 4.1: a single clean galaxy whose hidden component turns over at $r > 3L_h$ or $r < L_h/3$ ends the one-length universality the kernel reading requires, and the residual masked core–disk slope ($0.254 \pm 0.110$) must vanish as samples improve.

**The cokernel.** The typing of dark energy stands or falls with P32: a confirmed departure of $\Omega_\Lambda$ from $\Lambda_0 = 1-\pi^2/32$ beyond the corner-residual structure removes the cokernel half of Theorem 4.1, leaving the kernel half intact but the unification claim broken.

# 8 Relation to the Corpus

This paper consolidates and types; it does not introduce mechanism. Its position in the corpus is as follows.

*Upstream.* P27 and P34 supply the constraint $C \circ P = I$ whose formal type Definition 2.1 fixes; Remark 2.3 closes P27's open remark P027_2. P32 supplies $\Lambda_0 = 1 - \pi^2/32$, consolidated here as the cokernel half of the typing. P37 supplies the irrationality of the rotation number that drives Corollary 3.5. A254 supplies the collapse delta, hereby the defect functor of the topological ambient. A259 supplies the $\ell(\ell + 1)$ bookkeeping of the charged sector; A269 supplies the pacing; A266–A268 supply the comma and locking instances.

*This paper.* A261's superselection theorem (sealing), A272's definition and typing (dark as kernel dynamics; kernel versus cokernel), and A276's family laws (composition, the kernel ladder, dark dominance at depth) are assembled into one statement: the two cosmological darks are the two exactness defects of one map family, and the kernel is sealed, paced, and never empty.

*Downstream.* Paper 39 consumes the kernel as the measured halo: its profile, amplitude law, and SPARC confrontation presuppose Corollaries 3.3 and 3.4. The operating system consumes Proposition 5.7: luminosity semantics, plateau detection as kernel-rank estimation, and the Butlerian line as kernel protection. The 18-sector bridge (Pin 5) locates the fold as a Weld-sector event, which is where Remark 5.3 deposits the defect's energy.

# 9 Conclusion

Dark is not a substance; it is a defect of exactness. One map family, the split idempotents of the corpus's closure data, has two defects: what the projection annihilates and what the section fails to cover. The first gravitates, keeps the canonical clock, and cannot convert, and is dark matter; the second is a measure, $1 - \pi^2/32$, and is dark energy. The kernel is sealed by superselection, paced by the shared breath, and never empty by irrationality; composition of closures can only grow it, so dark dominance at depth is a theorem, not a coincidence. The same formalism runs the operating system, where dark entries are kernel states and the refusal to silently promote them is the computational face of the same superselection that keeps the halo dark. The discovery channel this account nominates is an absence: a band in which the canon predicts silence, so that any signal found there is, by construction, news.

**Provenance and verification.** Assembled without new claims from A261 (`verify_P261`, 16 checks), A272 (`verify_P272`, 11), A276 (`verify_P276`, 12), all green at filing. Cross-references: A254, A255, A256, A257, A259, A260, A264, A266–A269, A289, P27, P32, P34, P37, P39.