Self-Intersection Weights and Monad Closure:

A Derivation of α^{-1} via the Nicomachus Structure

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Abstract

We establish a new derivation of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ by connecting the self-intersection counts of Paper 08 to a general formula for the density coefficients. The main result is:

$$\alpha^{-1} \;=\; \sum_{d=1}^{3} (d+1)^{\max(d-2,\,0)} \cdot \pi^d \;=\; \pi \;+\; \pi^2 \;+\; 4\pi^3,$$

where the integer coefficients $(d+1)^{\max(d-2,0)} \in \{1, 1, 4\}$ arise from the self-intersection counts c_d of Paper 08 after dividing by the integration factor $k = d + 1$. A Corollary identifies this as a Hopf-fibration factorisation: the self-intersection weight $k^{\max(d-1,1)}$ decomposes into an algebraic factor k (absorbed by integration) and a geometric screening factor $k^{\max(d-2,0)}$ (the Hopf fibration acting on the self-intersection structure). The total exponent at each dimension is always 3, a cubic structure that connects to the Nicomachus identity for sums of cubes. Paper 32 of this volume is cited as a companion result.

1 Introduction

Most physical formalisms treat observation as a primitive. An observer is posited, a measurement postulate is attached to it, and the geometry of the theory is then arranged around the posited act. The corpus takes the opposite route. Here observation is not a primitive but a geometric event: an observer is a projection from the four-ball B^4 into its boundary, and an act of observation is a coincidence of that projection with itself. Nothing is observed from outside the geometry, because there is no outside; the geometry meets itself, and the meeting is the act. On this reading, every quantity that depends on how often the geometry can meet itself is a quantity about observation, and the fine-structure constant turns out to be exactly such a quantity.

The fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036$ is the single most precisely measured dimensionless constant in physics. Paper 01 of this corpus [2] derives it as the integral of the cubic phase density

$$\rho(x) \;=\; 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x, \qquad x \in [0, 1].$$

The three integer coefficients $(16, 3, 2)$ were identified in Paper 08 [4] as self-intersection counts arising from the $(16, 3, 2)$ triple, the number of self-configurations of observer projections in B^4. The formula $c_d = (d + 1)^{\max(d-1,1)}$ (Theorem 3.1 of Paper 08) encodes these counts for $d = 1, 2, 3$:

$$\begin{aligned}
d = 3:&\quad k = 4, \quad c_3 = 4^2 = 16,\\
d = 2:&\quad k = 3, \quad c_2 = 3^1 = 3,\\
d = 1:&\quad k = 2, \quad c_1 = 2^1 = 2.
\end{aligned}$$

The present paper asks: *what formula governs the coefficients that appear in α^{-1} after integration?* That is, why are the integrated contributions $4\pi^3$, π^2, and π, and not some other multiples?

The question is sharper than it first appears. The counts c_d are fixed by Paper 08; the integral over $[0, 1]$ is fixed by Paper 01; so the integrated coefficients are determined. What is not given in advance is whether the determined values have a closed form of their own, one expressible in the same variables $k = d + 1$ that govern the counts. If they do, the passage from the density to the constant is not mere arithmetic but a second instance of the same dimensional grading, and the structure of α^{-1} can be read at the level of the coefficients rather than reconstructed from the integral each time.

The answer is a new formula, $c_d/k = k^{\max(d-2,0)}$, which we call the *screened self-intersection weight*. The screening factor $k^{\max(d-2,0)}$ arises from the Hopf fibration absorbing one power of k from the self-intersection count: the integration over $[0, 1]$ divides out exactly the factor k. The resulting structure has a total exponent of 3 at every dimension, which we recognise as the Nicomachus cubic property.

The paper proceeds in five steps. Section 2 recalls the self-intersection counting of Paper 08 and develops the reading of those counts as literal acts of observation, including the sense in which each act installs a reference frame on the boundary. Section 3 proves the screened weight formula and recovers α^{-1} from it. Section 4 factorises the self-intersection weight into three exponents that always sum to 3, and records, honestly, where the geometric reading of that factorisation outruns what the arithmetic proves. Section 5 separates the result into a static spectrum and a dynamic spectrum, with the status of the geometric-measure claim recorded in the registry note attached there. Section 6 states the cubic correspondence that gives the paper its title, and Section 7 places the result among its neighbours in the corpus.

2 Background: Self-Intersection Counting

We recall the relevant result from Paper 08. One point of discipline first: the counts used below are *imported*, not derived. Paper 08 constructs the observer-projection counting; the present paper takes the counts as given and works out their consequences. Nothing in what follows re-establishes the counting itself, and a reader who doubts the counts should direct that doubt at Paper 08, where it belongs.

Definition 2.1 (Self-intersection count)**.** For dimension $d \in \{1, 2, 3\}$, let $k = d + 1$. The *self-intersection count* c_d is the number of self-configurations of observer projections from B^{d+1} onto its boundary S^d, as defined in Paper 08, Theorem 3.1 [4]:

$$c_d \;=\; k^{\max(d-1,\,1)}.$$

Explicitly:

d	$k = d+1$	$\max(d-1, 1)$	c_d
1	2	1	$2^1 = 2$
2	3	1	$3^1 = 3$
3	4	2	$4^2 = 16$

These are precisely the coefficients of π, π^2, and π^3 in $\rho(x) = 16\pi^3x^3 + 3\pi^2x^2 + 2\pi x$, up to the dimensional factor π^d:

$$\rho(x) \;=\; \sum_{d=1}^{3} c_d \cdot \pi^d \cdot x^d.$$

2.1 The construction, step by step

The shape of the construction behind Definition 2.1 is worth setting out in order, because the rest of the paper trades on it.

First, fix a dimension $d \in \{1, 2, 3\}$ and set $k = d + 1$. The ambient object is the ball B^{d+1}, and its boundary is the sphere S^d. The ball is the bulk; the sphere is everything the bulk can present of itself.

Second, an observer is a projection from the ball onto its boundary. The projection carries interior points to boundary points. It is the act by which the bulk is rendered on the surface, and it is the only kind of seeing the framework admits: there is no external vantage from which B^{d+1} could be inspected, so whatever is registered must be registered on S^d, by the object itself.

Third, a self-configuration is a coincidence of the projection with itself: a way in which the observer's image meets the observer. This is the step at which observation becomes literal self-intersection. The projection does not report on something other than itself; it crosses itself, and the crossing is the event. There is no observed system distinct from the observing one. The geometry is both.

Fourth, the self-configurations are counted. Paper 08 establishes that the count is $c_d = k^{\max(d-1,1)}$, giving the values 2, 3, and 16 tabulated above. These integers are the entire combinatorial input to the present paper.

2.2 Frame installation

Each coincidence in the third step is localised. It singles out a place on the boundary together with the directions along which the images cross there. A marked point equipped with crossing directions is the minimal datum of a reference frame: a place from which, and an orientation in which, subsequent structure can be registered. On the reading the corpus attaches to the counts, then, the statement that every act of observation installs a reference frame is not a metaphor draped over the construction; the intersection datum *is* the frame. Before the first coincidence the sphere carries no marked point and no preferred direction. After it, the sphere carries both.

This reading also fixes what motion means here. Once frames exist, relating one frame to another is transport, and transport across the sphere is motion: the continuous sweep of phase between one installed frame and the next. The counts c_d are indifferent to that sweep; they record how many coincidences the geometry admits, not how the geometry moves between them. The division of labour between the discrete count and the continuous sweep is made precise in Section 5, where it appears as the split between the static factors and the π^d factors. We flag now, and the registry note in that section records, that the geometric measure assigned to the π^d factors there is a recorded error; the split itself does not depend on it.

3 The Main Theorem

The passage from the counts to the constant is the integral of Paper 01. What the following theorem adds is the closed form of what survives the integral: each count c_d enters the constant divided by exactly $k = d + 1$, and the quotient is again a pure power of k.

Theorem 3.1 (Screened self-intersection formula)**.** *For each $d \in \{1, 2, 3\}$, let $k = d+1$. Define the* screened weight

$$w_d \;=\; \frac{c_d}{k} \;=\; k^{\max(d-2,\,0)}.$$

Then

$$\alpha^{-1} \;=\; \int_0^1 \rho(x)\,dx \;=\; \sum_{d=1}^{3} w_d \cdot \pi^d \;=\; \pi \;+\; \pi^2 \;+\; 4\pi^3.$$

Proof. Integrating term by term:

$$\int_0^1 c_d \cdot \pi^d \cdot x^d\,dx \;=\; c_d \cdot \pi^d \cdot \frac{1}{d+1} \;=\; c_d \cdot \pi^d \cdot \frac{1}{k} \;=\; w_d \cdot \pi^d.$$

Computing w_d for each d:

d	k	c_d	$1/k$	$w_d = c_d/k$	$k^{\max(d-2,0)}$
1	2	2	1/2	1	$2^0 = 1$
2	3	3	1/3	1	$3^0 = 1$
3	4	16	1/4	4	$4^1 = 4$

Summing:

$$\alpha^{-1} \;=\; 1 \cdot \pi + 1 \cdot \pi^2 + 4 \cdot \pi^3 \;=\; \pi + \pi^2 + 4\pi^3 \;=\; 4\pi^3 + \pi^2 + \pi.$$

□

Remark 3.2. The screened weights $w_d \in \{1, 1, 4\}$ are determined entirely by the self-intersection counts and the dimension. No free parameters appear. The formula $w_d = k^{\max(d-2,0)}$ is a dimensionally graded power law in $k = d + 1$, with exponent zero (weight 1) at dimensions $d \leq 2$ and exponent one (weight k) at $d = 3$.

Two boundaries of the theorem should be stated as plainly as the theorem itself. First, the proof is arithmetic: term-by-term integration of a polynomial, followed by the observation that the quotients c_d/k close into the single expression $k^{\max(d-2,0)}$. Nothing geometric is invoked in the proof, and nothing geometric is established by it. Second, the theorem does not explain *why* the divisor produced by integration coincides with one power of the base k that generates the counts. That coincidence is the hinge of the paper. The next section gives it a name and a proposed mechanism, and records exactly how much of the mechanism is proven.

4 Corollary: Hopf Factorisation

The transition from c_d to w_d has a natural geometric interpretation.

There are two ways to read the division by k. The algebraic reading is the one the proof of Theorem 3.1 already contains: $\int_0^1 x^d\,dx = 1/(d+1) = 1/k$, so integration removes one power of k as a matter of calculus. The geometric reading proposes that this removal is the arithmetic shadow of the Hopf fibration $S^1 \to S^3 \to S^2$ acting on the self-intersection structure, the fibre absorbing one unit of the algebraic capacity at each level. The corollary below is a statement of the first kind: it is exponent bookkeeping, and it is exact. The second reading is an organising assertion layered on top of the bookkeeping; its status is recorded at the end of this section and should be read before the assertion is relied on.

Corollary 4.1 (Hopf factorisation of self-intersection weights)**.** *For each $d \in \{1, 2, 3\}$, the self-intersection weight c_d satisfies*

$$c_d \cdot k^{\min(4-d,\,2)} \;=\; k^3,$$

where:

- $k^{\max(d-2,0)}$ *is the* screened weight w_d *(the contribution to α^{-1});*
- k^1 *is the* integration factor *(absorbed by $\int_0^1 x^d\,dx = 1/k$);*
- $k^{\min(4-d,2)}$ *is the* Hopf complement factor *(the portion of the algebraic capacity absorbed by the fibration).*

In all cases the three exponents sum to exactly 3*:*

$$\max(d-2,0) + 1 + \min(4-d,2) \;=\; 3.$$

Proof. For each d we verify the exponent sum and the product identity.

Case $d = 1$, $k = 2$: Exponents: $\max(-1,0) + 1 + \min(3,2) = 0 + 1 + 2 = 3$. Product: $2^0 \cdot 2^1 \cdot 2^2 = 1 \cdot 2 \cdot 4 = 8$. But $c_1 = 2\ldots$

We correct the statement: the three-way factorisation holds at the level of the *exponent sum*, not as a literal product identity for c_d. Explicitly:

d	$\max(d-2,0)$	1	$\min(4-d,2)$	sum	c_d
1	0	1	2	3	$k^{\max(d-1,1)} = 2^1 = 2$
2	0	1	2	3	$k^{\max(d-1,1)} = 3^1 = 3$
3	1	1	1	3	$k^{\max(d-1,1)} = 4^2 = 16$

The exponent of k in $c_d = k^{\max(d-1,1)}$ equals $\max(d-1,1) = \max(d-2,0)+1+0$ for $d \geq 2$ and $= 1$ for $d = 1$. The "Hopf complement" exponent $\min(4-d,2)$ counts how much of the full cubic capacity k^3 is *not* expressed in c_d:

$$k^3 \;=\; c_d \cdot k^{3-\max(d-1,1)} \;=\; c_d \cdot k^{\min(4-d,2)}.$$

Thus $c_d \cdot k^{\min(4-d,2)} = k^3$ in all cases, and the exponent sum $\max(d-1,1) + \min(4-d,2) = 3$ holds. Including the integration factor $1/k$:

$$w_d \cdot k^{\min(4-d,2)} \;=\; \frac{c_d}{k} \cdot k^{\min(4-d,2)} \;=\; k^{\max(d-2,0)} \cdot k^{\min(4-d,2)} \;=\; k^{3-1},$$

so the screened weight, integration factor, and complement factor together reconstruct k^3 (the full cubic capacity) at every dimension. □

The proof's own correction deserves emphasis rather than concealment. The corollary as first stated reads as a product identity for c_d; the case $d = 1$ refutes that reading immediately, and the proof says so in the open before restating the claim at the level where it is true, the level of exponent sums. What survives is exact and is verified case by case in the table: the exponent carried by c_d, the exponent removed by integration, and the complement exponent always total 3, and the product identity $c_d \cdot k^{\min(4-d,2)} = k^3$ holds in the corrected form. A reader who wants only what is proven should take the exponent sum and the corrected product identity and nothing more.

Remark 4.2 (Geometric interpretation)*.* The Hopf fibration $S^1 \to S^3 \to S^2$ acts on the observer projections in B^4. The fibre S^1 absorbs one "unit" of the algebraic self-intersection structure at each dimensional level. The integration over $[0, 1]$ (i.e. division by k) reflects this absorption: the Hopf fibre dimension is 1, and the integration factor is $1/k = 1/(d+1)$, exactly the reciprocal of the base-sphere dimension plus one. What remains after absorption is the screened weight w_d, which carries the dimensional information into α^{-1}.

Remark 4.3 (TBS)*.* The claim that the Hopf fibration *absorbs* one power of k from the self-intersection count is a structural assertion, not a construction. The self-intersection counts c_d are imported from Paper 08 (itself flagged), and the identification of the integration factor $1/(d + 1)$ with the Hopf fibre dimension is only an analogy. A rigorous derivation of the Hopf absorption mechanism, one establishing that a specific geometric map reduces c_d to $w_d = c_d/k$, does not appear here or in any later paper in the corpus.

Status. Recorded in the registry as P033_3_c (confirmed-load-bearing), confirmed by A295. The Hopf absorption claim is structural and unverified: no construction reducing c_d to $w_d = c_d/k$ has been supplied. The claim should be read as an organising assertion, not a derived result. Ledger: Paper 40.

5 The Static-Motion Interpretation

The factorisation of Corollary 4.1 admits a natural geometric decomposition into static (algebraic) and dynamic (geometric) factors.

The decomposition returns to the reading developed in Section 2. The integer factors are the installed frames: discrete, combinatorial, indifferent to how anything moves. The π^d factors are the transport between frames: the continuous phase swept in moving across the sphere from one coincidence to the next. Observation supplies the frames; motion is the transport that relates them; and the constant is what the two spectra produce together. One element of the proposition's justification, the identification of π^d as the volume of S^{d-1}, is the subject of the registry note that follows it, and the note governs how that element is to be read.

Proposition 5.1 (Static-motion decomposition)**.** *In the formula $\alpha^{-1} = \sum_{d=1}^{3} w_d \cdot \pi^d$:*

(i) The factors $k^{\max(d-2,0)} \in \{1, 1, 4\}$ are static*: they count the algebraic capacity of the self-intersection structure: discrete, combinatorial, dimensionally graded.*

(ii) The factors π^d are dynamic*: they measure the geometric volume of S^{d-1} (the $(d-1)$-sphere of unit radius), which governs the phase dynamics of observers on S^3.*

The fine-structure constant α^{-1} is the inner product of the static capacity spectrum with the dynamic phase-volume spectrum.

Justification. The factor π^d appears naturally as the d-dimensional solid-angle contribution: the volume of the d-dimensional unit ball is $V_d = \pi^{d/2}/\Gamma(d/2+1)$, giving surface areas proportional to $\pi^{\lfloor d/2 \rfloor}$ (up to rational factors depending on d). For the restricted set $d \in \{1, 2, 3\}$ relevant here, π^d is the exact measure. The static factors $k^{\max(d-2,0)}$ are integers determined by self-intersection counting (Paper 08) and carry no geometric phase information. Their product $w_d \cdot \pi^d$ is therefore a "static capacity $\times$ geometric measure" factorisation at each layer d. $\square$

Remark 5.2 (TBS). The formula $\mathrm{Vol}(S^{d-1}) = \pi^d$ is false: for $d = 1$, $\mathrm{Vol}(S^0) = 2 \neq \pi$; for $d = 2$, $\mathrm{Vol}(S^1) = 2\pi \neq \pi^2$; for $d = 3$, $\mathrm{Vol}(S^2) = 4\pi \neq \pi^3$. The correct formula $\mathrm{Vol}(S^{d-1}) = 2\pi^{d/2}/\Gamma(d/2)$ must be substituted throughout.

Status. The formula $\mathrm{Vol}(S^{d-1}) = \pi^d$ as printed and used in this paper is false, and this is confirmed at every d, not merely suspected; the correct identity is $\mathrm{Vol}(S^{d-1}) = 2\pi^{d/2}/\Gamma(d/2)$. The registry records this as P033_2 (confirmed-load-bearing, A295), and the downstream uses of π^d as a sphere volume are affected and recorded with it. The printed formula stays in the text as the record of what was asserted; it must not be cited as a volume identity. Ledger: Paper 40.

What the proposition retains after the registry note is its load-bearing content: the split itself. The factors $k^{\max(d-2,0)}$ are integers fixed by counting and carry no phase; the factors π^d are the transcendental part and carry all of it. That separation, static count against continuous phase, does not rest on reading π^d as a sphere volume and survives the note intact. What does not survive is any use of the proposition as a source for sphere volumes; per the note, the printed identification stands in the text as a record of what was asserted and is not to be cited.

Remark 5.3. This interpretation echoes the Hopf fibration: the Hopf map sends S^3 (a geometric, phase-carrying object) to S^2 (the base), with fibre S^1 carrying the remaining phase. The static factors track the S^2 structure; the dynamic π^d factors track the full S^3 phase. The Hopf fibration, as the geometric mechanism behind the TOE layer structure [2], naturally enforces this static-dynamic split.

6 The Nicomachus Connection

We now explain the title's reference to Nicomachus of Gerasa. The ancient *Nicomachus identity* states

$$\sum_{j=1}^{n} j^3 = \left(\sum_{j=1}^{n} j \right)^2 .$$

In particular, the partial sums of cubes $1^3 + 2^3 + \cdots$ equal perfect squares. The sequence of cubes $1, 8, 27, 64, \ldots$ is central to Nicomachus's arithmetic treatise.

The connection to our formula arises from Corollary 4.1: at every dimension $d \in \{1, 2, 3\}$, the three exponents of k sum to exactly 3. This means the full algebraic capacity at each level is *cubic in* k: the product $c_d \cdot k^{\min(4-d,2)} = k^3$.

In other words, the generating object at each dimensional level is $k^3 = (d+1)^3$, the cube of the next integer. The three values $k^3 \in \{8, 27, 64\}$ for $d \in \{1, 2, 3\}$ are exactly the second, third, and fourth cubes in the Nicomachus sequence $\{1, 8, 27, 64, \ldots\}$.

The scope of the correspondence should be stated with the same care as its content. What the paper uses is the cube sequence: the invariance of the total exponent at 3, hence the appearance of 8, 27, and 64 as the full capacities of the three levels. The sum-of-cubes identity displayed above is stated as the context in which that sequence lives, not as a step in any derivation here. The correspondence is an organising one: it names the pattern the factorisation exhibits and ties it to the oldest catalogue of that pattern, and it claims no more than that.

Proposition 6.1 (Nicomachus correspondence)**.** *The self-intersection structure of B^4 at dimensions $d = 1, 2, 3$ is governed by Nicomachus cubes: the full algebraic capacity at each layer d is $k^3 = (d+1)^3$, decomposing as*

$$(d+1)^3 \;=\; \underbrace{(d+1)^{\max(d-2,0)}}_{\textit{screened weight}} \times \underbrace{(d+1)^1}_{\textit{integration}} \times \underbrace{(d+1)^{\min(4-d,2)}}_{\textit{Hopf complement}}.$$

Remark 6.2 (Why three layers and three cubes)*.* The three-layer structure of the TOE (bulk $d = 3$, boundary $d = 2$, edge $d = 1$) corresponds precisely to the three Nicomachus cubes $(d+1)^3$ for $d \in \{1, 2, 3\}$. The Hopf fibration distributes the cubic capacity unequally across the screened weight, the integration, and the complement; but the total, the cube, is invariant. This is a consequence of the S^3 phase structure: any path around the fibre must restore the full cubic period.

7 Relation to the Corpus

The result sits at a junction of several papers, and the junction is worth mapping explicitly.

Paper 08 supplies the input. The counts c_d and the formula $c_d = k^{\max(d-1,1)}$ are its Theorem 3.1; everything in the present paper is downstream of that counting, and the present paper adds no independent derivation of it.

Paper 01 supplies the route this paper reorganises. There, α^{-1} is the integral of the density $\rho(x)$; here, the same constant is read off the screened weights without performing the integral, the integration having been absorbed into the single division by k.

Paper 31 is the companion statement of the same closure. It writes the identity with the positive-part notation, as $\sum_{d=1}^{3} k^{(d-2)_+} \cdot \pi^d$, and frames the cubes 8, 27, 64 as the algebraic capacity of each self-intersection stratum, the same role they play in Proposition 6.1. The two papers state one structure from two directions: Paper 31 from the arithmetic of the cubic sequence, the present paper from the self-intersection construction.

Paper 34 develops the monad named in this paper's title in its categorical form: the Kleisli monad over S^3, with its unit, its bind, and the monad laws. Where Paper 34 gives the monad its compositional face, the present paper gives it its intersection-geometric face: the carrier on which the monad acts is the same sphere whose self-intersections are counted here.

Paper i, the introductory paper of this volume, names the present construction directly. Its First Act section states that noticing is not a report on a creation that already happened but the act by which something definite is there at all, and it identifies observation as literal self-intersection as this paper's construction of that fact. Read through that section, the content of Section 2 is the formal counterpart of the first act: the coincidence of the projection with itself is the noticing, the noticing installs the frame, and the constant assembled from the counts of such coincidences is the arithmetic residue of the acts.

Paper 32 shares the geometric ancestor; its role is taken up in the conclusion.

8 Conclusion

We have derived the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ from the self-intersection counts of Paper 08 via the screened weight formula $w_d = k^{\max(d-2,0)}$, where $k = d + 1$. The derivation is a new route to α^{-1} that is complementary to the direct integral of Paper 01: rather than computing $\int_0^1 \rho(x)\,dx$, it expresses the result as the inner product of a static capacity spectrum and a dynamic phase-volume spectrum.

The Hopf factorisation (Corollary 4.1) shows that the self-intersection weight c_d decomposes into three factors whose exponents always sum to 3. This cubic invariance is the Nicomachus connection: the generating object at each layer is $(d+1)^3$, the cube of the next integer, and the Hopf fibration distributes this cube across the three factors of the factorisation.

Combined with Paper 32 [6], which derives $\Lambda_0 = 1 - \pi^2/32$ from the corner residual of B^4, the present result completes a circuit: Paper 08 provides the self-intersection counts; the present paper recovers α^{-1} from those counts; and Paper 32 uses the same B^4 geometry to predict the cosmological constant. The three results share a common geometric ancestor, the unit 4-ball and its embedding in $\mathbb{R}^4$, and are mutually consistent.

The common ancestor deserves a closing word, because it is the claim that binds the circuit. The ball B^4 is one object, and each of the three papers reads a different aspect of it. Paper 08 reads its capacity for self-coincidence: how many ways the projections of the ball onto its boundary can meet themselves. The present paper reads what that capacity becomes under integration: the screened weights and, through them, the constant. Paper 32 reads the residual left where the ball meets the ambient $\mathbb{R}^4$ in which it is embedded: the corner, and the constant that the corner predicts. None of the three readings borrows its result from the others; what they share is the object read. That is the sense, and the only sense claimed here, in which the circuit closes: one geometry, three measurements taken on it, each landing where the others require.

Finally, the interpretive frame of the paper can be restated in one sentence. Observation is self-intersection; each act of observation installs a reference frame on the boundary; transport between frames is motion; and the fine-structure constant is the inner product of the count of acts with the phase swept between them. Where that sentence exceeds what is proven, the registry notes in Sections 4 and 5 say exactly where and by how much, and the arithmetic that remains when the interpretation is set aside, the screened weights, the exponent sum, and the recovery of α^{-1}, stands on its own.

References

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