Self-Referential Boundary Inversion and the Quintic Structure of the Fine-Structure Constant

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2025-12

Abstract

We derive the irreducible quintic fixed-point equation governing the emergence of the fine-structure constant α from the inversion of the self-referential observation operator defined in the geometric (B^4, S^3, S^1) framework. Previous papers established the energy equilibrium $E_{\text{self}} = 13.177$, the oscillation law $\kappa = \alpha^{5/4}$, and the spectral correspondence between geometric and quantum beta functions. However, these results treated α as an external normalization parameter. In this work, we reverse the direction: we treat α as *unknown* and impose the three global consistency conditions implied by the framework. We prove that these constraints collapse to a single fifth-degree algebraic equation whose unique positive real root is the physical α. This paper completes the missing structural link between the spectral papers and the fermion-sector mass papers. The corpus's standing diagnosis (Addendum 293, Paper 40) is that what follows is a schema rather than a completed construction: the consistency conditions and the elimination map are not specified to the point where the quintic can be computed or its claims verified.

Contents

1 Introduction

The geometric framework established across *Mathematical Foundations of Geometric Fundamental Physics*, *The Perfect Stable Sphere*, *Geometric Spectral Theory*, and *Self-Referential Observation and the Fermion Sector* determines much of low-energy physics from three irreducible geometric layers: bulk B^4, boundary S^3, and observational S^1 fibers.

These works showed that the cubic density

$$\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$$

satisfies

$$\int_0^1 \rho(x)\,dx = 4\pi^3 + \pi^2 + \pi \approx 137.036,$$

reproducing α^{-1} to 0.0002% precision.

But one major structural element remained unproven:

What determines α itself?

The answer is sought in the inversion of the self-referential observation operator. Here we set out how this inversion would lead to an irreducible *quintic* in α; as the abstract and the Status note record, what follows is a schema for that construction, not the construction itself.

2 Background and Prior Results

2.1 The cubic density and its moments

The cubic density generates moments

$$\mu_n = \int_0^1 x^n \rho(x)\,dx = \frac{16\pi^3}{n+4} + \frac{3\pi^2}{n+3} + \frac{2\pi}{n+2}.$$

Key values:

$$\mu_0 = \alpha^{-1}, \qquad \mu_1 \approx 108.71668, \qquad \mu_2 \approx 90.17596.$$

2.2 Self-lensing energy and oscillation parameter

The self-lensing energy functional

$$E_{\text{self}} = \frac{\int_0^1 (\rho'(x))^2 dx}{2\mu_0^2}$$

evaluates to 13.177, creating the oscillation arena described previously.

The oscillation parameter satisfies

$$\kappa = \alpha^{5/4}.$$

2.3 The observation operator

The operator is

$$\hat{O}_\alpha = -\frac{d^2}{dx^2} + \frac{(\rho'(x))^2}{2\mu_0^2} + E_{\text{self}}\, x^2(1-x)^2.$$

Its first eigenvalue $\lambda_1(\alpha)$ generates the electron mass scale.

3 Forward vs. Inverse Problem

The previous papers solve the *forward* problem:

$$\alpha \to \rho(x) \to \mu_n \to V(x;\alpha) \to \hat{O}_\alpha \to \lambda_1 \to m_e.$$

Here we solve the *inverse* problem:

$$m_e \to \lambda_1(\alpha) \to \hat{O}_\alpha \to V(x;\alpha) \to \mu_0(\alpha) \to \alpha.$$

This inversion creates the quintic structure.

4 Deriving the Quintic Consistency Equation

4.1 Condition 1: κ-law

From $\kappa = \alpha^{5/4}$ and

$$\kappa_{\mathrm{geom}}(\alpha) = C_1 \frac{\mu_2}{\mu_0^2} + C_2 \frac{\mu_1}{\mu_0} + C_3,$$

we equate:

$$\kappa_{\mathrm{geom}}(\alpha) = \alpha^{5/4}.$$

Raising to the fourth power eliminates fractional exponents:

$$\kappa_{\mathrm{geom}}(\alpha)^4 = \alpha^5.$$

4.2 Condition 2: eigenvalue constraint

Let $F(\lambda_1(\alpha), \beta(\alpha)) = 0$ encode the electron mass scale.

Both λ_1 and $\beta = \mu_1/\mu_0$ depend on α rationally.

Remark 1 (TBS). *The claim that $\lambda_1(\alpha)$ depends rationally (or even algebraically) on α is asserted without proof. The first eigenvalue of the Sturm–Liouville operator $\hat{O}_\alpha = -d^2/dx^2 + V(x;\alpha)$ with α-dependent potential is a transcendental function of α in general; establishing any algebraic dependence requires a separate theorem that is not provided here or elsewhere in the corpus.*

4.3 Condition 3: normalization

$\mu_0 = \alpha^{-1}$.

4.4 Consolidation

After algebraic elimination of intermediate quantities:

$$N(\alpha)^4 - \alpha^5 D(\alpha)^4 = 0,$$

where N and D are polynomials in α from the moment structure.

Clearing denominators yields:

$$P(\alpha) = a_5\alpha^5 + a_4\alpha^4 + a_3\alpha^3 + a_2\alpha^2 + a_1\alpha + a_0 = 0.$$

Remark 2 (TBS). *The claim that denominator-clearing of the three consistency conditions yields exactly degree 5 rather than a higher degree is asserted but not proved; a complete Gröbner-basis or resultant computation demonstrating the degree bound is required.*

This is the quintic.

Remark 3 (TBS). *The coefficients $a_5, a_4, a_3, a_2, a_1, a_0$ of the claimed irreducible quintic are never explicitly computed from the three consistency conditions; only their existence is asserted. The explicit coefficient expressions must be derived and verified.*

Status. Registry items P013_1, P013_2, and P013_3_c are all confirmed load-bearing (A293), and together they constitute the corpus's standing diagnosis of this paper, also stated in Paper 40: the quintic structure is a schema, not a construction. Conditions 1 through 3 and the elimination map F are unspecified, so the coefficients $a_5, \ldots, a_0$ are uncomputable as stated and the degree-5 bound is unverifiable until the construction is supplied. The rationality premise behind Condition 2 is unproven, and is generically false for Sturm–Liouville eigenvalues. The schema stands as the record of the intended structure; none of its quantitative claims is established. Ledger: Paper 40 and addenda/verify/tbs_registry.json.

5 Irreducibility

If the construction were completed as claimed, with explicit coefficients $a_5, \ldots, a_0$ computed from Conditions 1–3 and the elimination map (see the Status note above), the intended argument would run:

- no rational root would exist, by the Rational Root Test applied to the computed coefficients;
- the Galois group would be shown to be S_5, implying insolubility by radicals;
- the coefficients would reflect the five geometric contributions:

(1) bulk term x^3,
(2) boundary term x^2,
(3) edge term x,
(4) self-lensing correction,
(5) oscillatory exponent $5/4$.

None of these steps can currently be carried out: the coefficients are uncomputable as stated (Remark 3), so neither the Rational Root Test nor any Galois-group computation can be performed. The claim that the quintic is geometrically irreducible is conditional on completing the construction.

6 Physical Interpretation

The quintic structure reflects:

1. Five-layer constraint network.
2. Five degrees of curvature freedom.
3. Five distinct contributions to boundary feedback.
4. Non-reducibility of the three-layer ontology extended by two dynamical laws.

The number 5 appears because the feedback loop has five independent terms.

7 Numerical Verification Program

To confirm α, once the construction is supplied:

1. Compute $P(\alpha)$ numerically near $\alpha^{-1} = 137.036$.

2. Use Newton iteration on $P(\alpha)$.
3. Verify uniqueness of positive root.
4. Cross-check with full eigenvalue computation.

None of these steps can be executed as the paper stands: until Conditions 1–3 are made explicit and the constraint F is supplied, there is no computable $P(\alpha)$ to evaluate, no polynomial for Newton iteration, no coefficients against which to isolate the root, and no eigenvalue problem to cross-check. The program is the verification protocol for the completed construction, not verification that has been performed.

8 Conclusion

We have proposed a schema for the structural equation of the geometric framework: if the three consistency conditions and the elimination map were supplied, the inversion of the self-referential observation operator would determine α as the positive root of a quintic.

Per the standing diagnosis (Addendum 293, Paper 40), the schema is not yet a construction. The coefficients $a_5, \ldots, a_0$ are uncomputable as stated, the degree-5 bound is unverified, and the rationality premise behind Condition 2 is unproven. Completing the construction, by specifying Conditions 1–3 and the constraint F, computing the coefficients, and establishing the degree and irreducibility claims, is the open task this paper leaves to the corpus; only then would the theoretical circle begun in the previous papers close.

References

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