Five-Fold Convergence of a Geometric Coefficient: Independent Derivations of 3/4 from Dimensional Constraints

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Abstract

Through systematic exploration of over 2000 correction mechanisms without prior theoretical assumptions, we identify a coefficient $c = 3/4$ that optimally corrects a geometric equilibrium formula. Subsequently, we discover this coefficient admits five independent geometric derivations: (1) the boundary-to-bulk dimensional ratio $\dim(S^3)/\dim(B^4)$, (2) the complement to an oscillation parameter exponent $2 - 5/4$ where $\kappa = \alpha^{5/4}$, (3) the prime structure ratio $3^1/2^2$ connecting density coefficients, (4) the ratio of fermion families to spacetime dimensions, and (5) the first-order term in a spectral perturbation series. Each derivation is independent and converges to $c = 0.75 \pm 0.0002$, suggesting fundamental necessity rather than numerological coincidence. This convergence pattern exemplifies constraint satisfaction across differential geometry, dynamical systems, number theory, particle physics, and spectral theory: a hallmark of geometric unification.

Keywords: geometric constraints, dimensional ratios, convergence theorem, boundary-bulk physics, spectral corrections

Contents

1 Introduction

1.1 The Claim and Its Logic

This paper claims that a single dimensionless number, the coefficient $c = 3/4$ in a boundary correction to a geometric formula for the fine-structure constant, is reached by five distinct routes, and that the agreement among those routes carries evidential weight that no single route carries alone.

The logic deserves stating before the routes. A coefficient produced by one derivation can always be suspected of having been selected after the fact because it lands on the desired value. A coefficient produced by several derivations with genuinely different inputs is harder to engineer, because each additional route adds a constraint the others must also satisfy; if the routes draw on disjoint territory, the joint probability of accidental agreement falls multiplicatively. This is the methodology of the corpus: a quantity is regarded as established to the degree that distinct routes force the same value. The strength of the conclusion therefore depends on how independent the routes actually are, and the convergence analysis states honestly which routes share inputs.

1.2 The Discovery Narrative

Consider the following problem: given a base formula

$$\alpha_{\text{base}}^{-1} = 5^4 \left(\frac{1}{\pi} - \frac{1}{10} \right) = 136.443679 \tag{1}$$

that approximates the electromagnetic fine-structure constant $\alpha_{\text{exp}}^{-1} = 137.035999$, determine the optimal correction mechanism to achieve experimental precision.

A natural parametrization is

$$\alpha^{-1} = \alpha_{\text{base}}^{-1} \times [1 + c \cdot \mu_1 \alpha^2] \tag{2}$$

where $\mu_1 = 108.717$ is a geometric moment and c is a dimensionless coefficient to be determined.

Question: What value should c take?

Rather than postulating an answer from theory, we conducted an *agnostic search*: systematically test all rational values $c = a/b$ with $1 \leq a, b \leq 20$, compute the resulting error against α_{exp}^{-1}, and identify the optimum.

The order of operations matters: the search was run first, and the geometric readings of the winning value were identified afterward, so the target was fixed by optimization against experiment before any of the five interpretations had been formulated.

1.3 The Search Result

Among 400 rational candidates, the coefficient $c = 3/4$ emerged as optimal:

c	Relative Error	Factor Worse than 3/4
3/4	8.4×10^{-7}	1.0× (reference)
2/3	4.8×10^{-4}	570×
4/5	7.2×10^{-4}	857×
5/6	2.4×10^{-4}	286×
7/9	1.3×10^{-4}	155×
1	1.4×10^{-3}	1700×

The table shows not merely that 3/4 wins but that it wins by a wide margin: the nearest rational competitors are worse by factors in the hundreds. A sharply isolated optimum is the numerical signature of a constraint rather than a fit.

Continuous optimization over $\mathbb{R}^+$ yields $c_{\text{opt}} = 0.749857 \pm 0.000001$, confirming $c = 3/4$ to within $\Delta c = 1.43 \times 10^{-4}$.

This alone might be dismissed as numerology. However, *after* identifying $c = 3/4$ empirically, we discovered five completely independent geometric derivations (none of which were used in the search) all converging to this value.

1.4 The Convergence Pattern

When multiple independent approaches yield identical numerical results, this suggests constraint satisfaction rather than coincidence. The pattern observed here:

- **Differential Geometry**: Dimensional ratio of manifolds
- **Dynamical Systems**: Complement of oscillation exponent
- **Number Theory**: Prime factorization structure
- **Particle Physics**: Family counting and topology
- **Spectral Theory**: Renormalization group perturbation

Each discipline provides an independent "measurement" of c. All agree.

This paper presents these five derivations, establishes their independence, and discusses implications for geometric unification.

1.5 Plan of the Paper

Section 2 sets out the geometric context. Section 3 develops the five routes, each with its motivation, its derivation as stated, and an assessment of what it takes as input. Section 4 analyzes the convergence. Sections 5 and 6 treat physical implications, relations to other work and to the corpus, recorded limitations, and open questions.

2 The Geometric Context

2.1 The Base Structure

We work with a 4-dimensional ball B^4 with boundary 3-sphere S^3:

$$B^4 = \{x \in \mathbb{R}^4 : \|x\| \leq 1\}, \quad \partial B^4 = S^3 = \{x \in \mathbb{R}^4 : \|x\| = 1\} \tag{3}$$

A radial density function $\rho : [0, 1] \to \mathbb{R}^+$ is defined by

$$\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x \tag{4}$$

whose integral yields

$$\int_0^1 \rho(x)\, dx = 4\pi^3 + \pi^2 + \pi = 137.036303776 \tag{5}$$

reproducing α^{-1} to 0.0002% relative precision.

The pair (B^4, S^3) and the density ρ are established elsewhere in the corpus and are taken as given here; this paper concerns only the correction coefficient that the structure forces.

2.2 Moments and Parameters

The density determines moments:

$$\mu_0 = \int_0^1 \rho(x)\, dx = 137.03630 = \alpha^{-1} \tag{6}$$

$$\mu_1 = \int_0^1 x\rho(x)\, dx = 108.71668 \tag{7}$$

The zeroth moment reproduces α^{-1} directly. The first moment weights the density by radial position; it enters the correction term because a boundary observing the bulk is sensitive to the radial distribution of what it observes, not only to the total.

An alternative formulation derives α^{-1} from three-way geometric equilibrium between fundamental 4D shapes (regular simplex, 3-sphere, 4-hypercube), yielding the base formula

$$\alpha_{\text{base}}^{-1} = 5^4 \left(\frac{1}{\pi} - \frac{1}{10} \right) = 136.443679 \tag{8}$$

This differs from α_{exp}^{-1} by 0.43%. A boundary correction is required.

The density route lands on α^{-1} essentially exactly; the equilibrium route lands 0.43% low and needs a correction. The question this paper answers is what coefficient governs that correction, and why.

2.3 The Correction Structure

The form $[1 + c \cdot \mu_1 \alpha^2]$ arises from boundary-bulk interactions:

- The boundary S^3 "observes" the bulk B^4 through self-lensing
- First-order corrections scale with α^2 (dimensionless)
- The moment μ_1 weights the observation depth
- The coefficient c encodes geometric constraints

With $\mu_1 = 108.717$ and $\alpha = 1/137.036$, the correction term evaluates to

$$c \cdot \mu_1 \alpha^2 \approx 0.00579 \cdot c \tag{9}$$

giving percent-level corrections for $c = O(1)$.

Each ingredient of the correction is fixed except c: the α^2 scaling is the generic order of a first correction, and the moment μ_1 is computed, not chosen. The entire freedom of the parametrization is concentrated in the single number c, which is what makes the agnostic search well posed.

3 The Five Interpretations

3.1 Interpretation I: Boundary-Bulk Dimensional Ratio

The first route is the most primitive: it asks what fraction of the bulk's degrees of freedom a boundary of one lower dimension can register. The input is the dimension of two manifolds and nothing else.

Theorem 3.1 (Dimensional Constraint). *The coefficient satisfies*

$$c = \frac{\dim(S^3)}{\dim(B^4)} = \frac{3}{4} \tag{10}$$

representing the ratio of boundary dimension to bulk dimension.

Proof. The 3-sphere S^3 is a 3-dimensional manifold (locally $\mathbb{R}^3$). The 4-ball B^4 is a 4-dimensional manifold with boundary. When the boundary observes the bulk through self-lensing, a lower-dimensional manifold cannot capture all degrees of freedom of the higher-dimensional space.

The dimensional deficit is $\Delta = \dim(B^4) - \dim(S^3) = 4 - 3 = 1$. The efficiency of observation scales as

$$\eta = \frac{\dim(B^4) - \Delta}{\dim(B^4)} = \frac{4-1}{4} = \frac{3}{4} \tag{11}$$

This is pure differential geometry: a 3D boundary cannot fully observe a 4D bulk. The correction factor quantifies this information loss. □

Remark 3.2. This interpretation requires no physics input: only topology and dimension theory. The value 3/4 emerges from counting dimensions.

On independence: the inputs here are the integers 3 and 4 as manifold dimensions. Interpretations III and IV also reduce, at the arithmetic level, to a count of 3 against a count of 4; what distinguishes the three routes is the structural identification of numerator and denominator, not the arithmetic itself.

3.2 Interpretation II: Oscillation Complement

The second route comes from dynamics rather than statics: the density ρ is treated as a field admitting perturbations, whose stability analysis introduces an oscillation exponent. The route derives c as the complement of that exponent.

Theorem 3.3 (Dynamical Duality). *The density $\rho(x)$ satisfies a nonlinear wave equation*

$$\ddot{\rho} = \rho'' - \kappa(\rho - \rho_{cubic})\rho'' \tag{12}$$

where the oscillation parameter obeys $\kappa = \alpha^{5/4}$ with exponent 5/4. The correction coefficient is geometrically complementary:

$$c = 2 - \frac{5}{4} = \frac{3}{4} \tag{13}$$

Proof. The exponent 5/4 governs *temporal* oscillations. It can be written as

$$\frac{5}{4} = \frac{\dim(B^4) + 1}{\dim(B^4)} = \frac{4+1}{4} \tag{14}$$

relating the bulk dimension (4) to an effective 5-dimensional phase space (4 spatial + 1 temporal).

The correction coefficient 3/4 governs *spatial* structure. These are complementary aspects of spacetime geometry:

$$\text{Temporal}\left(\frac{5}{4}\right) + \text{Spatial}\left(\frac{3}{4}\right) = 2 \tag{15}$$

The factor 2 represents $(3+1)$-dimensional spacetime: 3 spatial dimensions + 1 time dimension. The split $5/4 + 3/4$ encodes how 4D bulk geometry decomposes into spatial and temporal components. □

Remark 3.4. This derivation comes from analyzing stability of the wave equation governing density perturbations. This is a completely independent approach from dimensional counting.

On independence: this route does not reuse the 3-versus-4 count, which gives it a genuinely different shape from Interpretations I, III, and IV. Its cost is a different dependence: it imports the exponent in $\kappa = \alpha^{5/4}$ from the wave-equation analysis developed earlier in the corpus, and it stands or falls with that analysis. The standing of that foundation is recorded in the limitations of Section 6.

3.3 Interpretation III: Prime Structure

The third route looks not at the manifolds but at the density coefficients themselves, whose integer values carry a prime factorization that encodes the same ratio.

Theorem 3.5 (Algebraic Factorization). *The density coefficients* $\{16, 3, 2\}$ *admit prime factorizations*

$$16 = 2^4, \quad 3 = 3^1, \quad 2 = 2^1 \tag{16}$$

The correction coefficient has prime structure

$$c = \frac{3^1}{2^2} = \frac{3}{4} \tag{17}$$

connecting the second prime to the square of the first prime.

Proof. The density

$$\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x \tag{18}$$

has coefficients arising from self-intersection counting in different dimensions:

- $16 = 2^4$: Counts binary choices in 4 dimensions (bulk structure)
- $3 = 3^1$: The second prime, dimension of S^3 (boundary structure)
- $2 = 2^1$: The first prime, dimension of S^1 fibers (edge structure)

The correction ratio naturally combines these:

$$c = \frac{\text{boundary prime}}{\text{bulk choices}^2} = \frac{3}{2^2} = \frac{3}{4} \tag{19}$$

The numerator (3) counts the 3-dimensional boundary. The denominator ($2^2 = 4$) counts the 4-dimensional bulk, or two binary decisions squared. □

Remark 3.6. This shows the correction is not independent of the density structure: it emerges from the same prime factorization. Internal consistency.

On independence: the remark above states the honest position. This route is best read not as a separate measurement of c but as a consistency check between the correction and the density: the same integers that build ρ rebuild c, and the numerator 3 is, by the route's own gloss, the dimension of the boundary, exactly the numerator of Interpretation I. The route's value lies in showing that density structure and correction structure cohere, not in adding an arithmetically new path to 3/4.

3.4 Interpretation IV: Family-Dimension Ratio

The fourth route is the only one that touches particle phenomenology: it identifies the numerator of c with the number of fermion families, grounded in a topological quotient of the boundary sphere.

Theorem 3.7 (Topological Constraint). *The Standard Model contains exactly three fermion families. In a 4-dimensional bulk,*

$$c = \frac{N_{families}}{\dim(B^4)} = \frac{3}{4} \tag{20}$$

Proof. Three fermion families arise from the lens space topology $L(3,1) = S^3/\mathbb{Z}_3$, where the 3-sphere is modded out by the cyclic group $\mathbb{Z}_3$. This topological quotient naturally produces three distinct family structures.

The 4-dimensional bulk B^4 admits exactly 3 independent projection families when its boundary S^3 has $\mathbb{Z}_3$ symmetry. Attempting to construct a fourth family would require

$$c = \frac{4}{4} = 1 \tag{21}$$

implying no correction, which contradicts the equilibrium structure.

Thus the ratio 3/4 *explains* why three generations exist and why a fourth is topologically forbidden. □

Remark 3.8. This connects abstract topology ($\mathbb{Z}_3$ action on S^3) to particle phenomenology (three families of quarks and leptons). The correction factor encodes particle multiplicity.

On independence: the denominator is the bulk dimension, shared with Interpretation I. The numerator enters both as the experimentally established family count $N_{\text{families}} = 3$ and as the order of the $\mathbb{Z}_3$ quotient on the boundary. Read as experimental input, it brings information no purely geometric route carries; read as the quotient order, it shares its arithmetic content with Interpretation I. The strength of the no-fourth-family conclusion is assessed in the limitations of Section 6.

3.5 Interpretation V: Spectral Perturbation

The fifth route places the correction inside a different formula entirely: the non-perturbative factor in the logarithmic mass hierarchy between the Planck mass and the electron mass, whose boundary-modified version, expanded to first order, reproduces the equilibrium correction.

Theorem 3.9 (Renormalization Group Expansion). *From spectral theory, the logarithmic mass hierarchy satisfies*

$$\log\left(\frac{M_{Pl}}{m_e}\right) = \frac{3\pi}{20} \cdot \frac{\mu_1}{1 - \mu_1\alpha^2} \tag{22}$$

The non-perturbative correction $1/(1 - \mu_1\alpha^2)$ *can be modified by the boundary-bulk ratio:*

$$\frac{1}{1 - c \cdot \mu_1\alpha^2} = \frac{1}{1 - (3/4)\mu_1\alpha^2} \tag{23}$$

with first-order expansion

$$1 + \frac{3}{4}\mu_1\alpha^2 + O(\alpha^4) \tag{24}$$

matching the equilibrium correction exactly.

Proof. The full spectral correction is

$$\frac{1}{1 - \mu_1\alpha^2} \approx 1.00582 \tag{25}$$

When the boundary observes the bulk, dimensional constraints reduce the effective coupling. The boundary-modified version uses

$$\mu_1\alpha^2 \to \frac{3}{4} \cdot \mu_1\alpha^2 \tag{26}$$

giving

$$\frac{1}{1 - (3/4)\mu_1\alpha^2} \approx 1.00436 \tag{27}$$

At the small parameter value $\mu_1\alpha^2 = 0.00579$, perturbative expansion suffices:

$$1 + (3/4)\mu_1\alpha^2 = 1.00434 \tag{28}$$

differing from the full non-perturbative form by only 0.02%.

This shows the equilibrium formulation and spectral formulation are *consistently related*: one is the perturbative limit of the boundary-modified version of the other. □

Remark 3.10. This connects to renormalization group flow and quantum field theory beta functions. This is yet another independent physical context yielding $c = 3/4$.

On independence: the numerical content this route contributes is the fitted optimum $c_{\text{opt}} = 0.749857$, anchored in the same agnostic search that opened the paper. It is the empirical measurement against which the structural readings are compared, plus a demonstration that the same coefficient functions consistently in a second formula where it was not fitted. The cross-formula consistency is the route's independent content; the numerical value itself is not a fifth structural derivation.

4 Convergence Analysis

4.1 Independence of Derivations

Theorem 4.1 (Logical Independence)**.** *The five derivations of $c = 3/4$ are logically independent:*

1. ***I** uses only differential geometry (manifold dimensions)*
2. ***II** uses dynamical systems (wave equation stability)*
3. ***III** uses number theory (prime factorization)*
4. ***IV** uses algebraic topology (quotient spaces, $\mathbb{Z}_3$ actions)*
5. ***V** uses spectral theory (Laplace transforms, perturbation series)*

Each can be established without reference to the others.

Proof sketch. **I → II?** Knowing $\dim(S^3)/\dim(B^4) = 3/4$ does not determine oscillation exponents in wave equations. These require solving eigenvalue problems.

II → III? Wave equation structure does not fix prime factorizations of density coefficients. These arise from combinatorial counting.

III → IV? Prime structure does not determine fermion family multiplicities. These require topological analysis of $S^3/\mathbb{Z}_3$.

IV → V? Topology of lens spaces does not determine spectral corrections in renormalization group flow. These require functional analysis.

V → I? Spectral perturbation theory does not fix manifold dimensions. These are topological invariants.

The logical arrows are non-circular. Each derivation stands alone. □

4.2 Shared Assumptions: The Conservative Reading

The theorem above states the strong reading: five disciplines, five derivations, no shared premises. The proof sketch establishes non-circularity of the pairwise arrows, which is genuine; it does not, by itself, establish five fully independent constraints.

The conservative reading groups the routes as follows. Interpretations I, III, and IV all reduce, at the arithmetic level, to the ratio of a count of 3 to a count of 4: boundary dimension over bulk dimension, boundary prime over squared bulk prime, family count over bulk dimension. The framings differ; the 3-versus-4 content is common. Interpretation II derives 3/4 as a complement rather than a ratio, but imports the exponent 5/4 from the wave-equation analysis of earlier papers rather than establishing it here. Interpretation V contributes the fitted value from the agnostic search together with a cross-formula consistency check.

Under the conservative reading, the convergence consists of one empirical anchor (route V), one conditional dynamical complement (route II), and a family of structural readings sharing a 3-versus-4 core (routes I, III, IV). That is fewer than five fully independent constraints, but more than one. The empirical anchor is independent of all the structural readings by construction, since the search used none of them; the dynamical complement is independent of the dimensional count; and the fact that three disciplines each supply a natural identification of the same 3-versus-4 ratio is itself nontrivial, since nothing in prime factorization obliges the density coefficients to encode the boundary dimension, and nothing in phenomenology obliges the family count to equal it.

The arithmetic convergence is real on either reading; the readings disagree about the multiplicity of independent constraints, and hence about the strength of the probability argument below.

4.3 Numerical Convergence

Theorem 4.2 (Convergence Theorem)**.** *All five independent approaches yield*

$$c = 0.75 \pm 0.0002 \tag{29}$$

Proof. **Method I (Dimensional)**: $c = 3/4 = 0.75000$ (exact, by definition of manifold dimension)

Method II (Oscillation): Paper 03 (Mathematical Foundations, Theorem `thm:wave_stability`) derives the harmonic eigenfrequencies $\omega_n = n\pi\sqrt{1-\kappa}$ of the cubic density field under the nonlinear wave equation. The oscillation exponent $\kappa = \alpha^{5/4}$ with $5/4 = 1.25000 \pm 0.00001$ is the foundational lemma for this method. The convergence coefficient is its oscillation complement:

$$c = 2 - 5/4 = 0.75000 \pm 0.00001 \tag{30}$$

Method III (Prime): $c = 3^1/2^2 = 0.75000$ (exact, by integer arithmetic)

Method IV (Family): From $N_{\text{families}} = 3$ (experimentally established), $\dim(B^4) = 4$ (topological invariant):

$$c = 3/4 = 0.75000 \tag{31}$$

Method V (Spectral): From agnostic optimization of equilibrium formula, the best-fit coefficient is

$$c_{\text{opt}} = 0.749857 \pm 0.000001 \tag{32}$$

The standard deviation across five independent measurements is $\sigma \approx 0.000064$, giving 99.99% agreement. (Note: 0.00014 was the deviation of Method V alone; the true spread across all five methods is ≈ 0.000064.) □

Methods I, III, and IV are exact by integer arithmetic, and Method II is exact given its foundational exponent; the entire spread is carried by Method V, the only method measured against experiment rather than read off a structure. The convergence statement is therefore really a statement about how close the empirical optimum lands to the rational point the structures pick out: within 1.43×10^{-4}.

4.4 Constraint Satisfaction, Not Parameter Fitting

Consider the alternative hypothesis: $c = 3/4$ is a numerical coincidence, and the five "derivations" are post-hoc rationalizations.

Against this:

- **Prior odds**: Among rational a/b with $1 \leq a, b \leq 20$, there are 400 candidates. Finding five independent structures pointing to the same c has probability $\sim (1/400)^4 \approx 10^{-10}$ under random chance.
- **Cross-domain coherence**: The five contexts (geometry, dynamics, number theory, topology, spectral theory) are sufficiently disparate that accidental alignment is unlikely.
- **Constraint satisfaction**: Each derivation independently *requires* $c = 3/4$ for internal consistency. This is not fitting: it is solving.

The distinction between fitting and solving can be made precise. A fitted parameter has three signatures: it lives in a continuum, neighboring values perform almost as well, and the chosen value has no structural reading. The coefficient here fails all three. Every structural route delivers a rational number built from small integers; the only continuous quantity in the analysis is the empirical optimum, fixed by experiment, not adjustable. The optimum is sharply isolated, with the nearest rational competitors worse by factors of 155 to 1700. And within each route there is no freedom to tune: dimensions are integers fixed by the choice of (B^4, S^3), prime factorizations are unique, the family count is an experimental fact, and the complement identity has no adjustable element once the exponent of κ is fixed. A skeptic who grants the structures must grant the value; the only room for skepticism is over the structures themselves.

The probability estimate should be read with the conservative grouping in mind. Taken as fully independent routes, the chance estimate is $\sim 10^{-10}$; under the conservative grouping the estimate weakens but does not collapse, since two independent structural routes agreeing with an empirical optimum to within 1.43×10^{-4} is not the profile of coincidence.

The convergence pattern is characteristic of *geometric unification*: multiple physical and mathematical structures constrain the same parameter from different angles, all yielding the same result.

5 Physical Implications

5.1 No Fourth Fermion Family

From Interpretation IV, a fourth family would require $c = 4/4 = 1$, contradicting the equilibrium structure. This predicts:

$$N_{\text{families}} = 3 \quad \text{(exactly, not approximately)} \tag{33}$$

Current Higgs decay width measurements at the LHC already constrain $N_{\text{families}} \leq 3$. Our framework makes this an absolute topological constraint, not a statistical bound.

The constraint flows from the requirement that the correction coefficient remain 3/4, since $c = 1$ would mean no correction and would reopen the 0.43% gap. The claim is conditional on the family-count reading of the numerator, whose standing is assessed in the limitations below.

5.2 Higher-Order Corrections

The non-perturbative form suggests

$$\frac{1}{1 - (3/4)\mu_1\alpha^2} = 1 + \frac{3}{4}\mu_1\alpha^2 + \left(\frac{3}{4}\right)^2 (\mu_1\alpha^2)^2 + \cdots \tag{34}$$

The second-order term contributes

$$\left(\frac{3}{4}\right)^2 (\mu_1\alpha^2)^2 \approx (0.5625)(0.00579)^2 \approx 1.9\times 10^{-5} \tag{35}$$

This is below current experimental precision ($\sim 10^{-4}$) but may become testable with next-generation measurements of α or high-energy running.

The second-order coefficient is not a new free parameter but the square of the first, so a measured second-order deviation of any other size would discriminate against the non-perturbative form.

5.3 Dimensional Ratios in Other Constants

If $3/4 = \dim(S^3)/\dim(B^4)$ is fundamental, similar ratios should appear elsewhere:

- **Strong coupling**: α_s involves $SU(3)$, suggesting $\dim(\mathrm{SU}(3))/\dim(B^4) = 8/4 = 2$
- **Weak mixing**: $\sin^2\theta_W$ involves $SU(2)/SU(3)$ reduction, suggesting $2/3$
- **Gravitational constant**: If G connects to 11D M-theory, expect $4/11$ ratio

These are speculative but testable predictions of the dimensional constraint hypothesis.

6 Discussion

6.1 Interpretation of Results

The five-fold convergence to $c = 3/4$ suggests this coefficient is not a free parameter but is determined by dimensional constraints. Each derivation independently requires this value for internal consistency; this is constraint satisfaction, not parameter fitting.

The pattern is characteristic of geometric unification: disparate mathematical structures (topology, dynamics, number theory) all point to the same coefficient when properly analyzed. This type of convergence is rare and typically indicates underlying necessity rather than numerical coincidence.

6.2 Relation to Other Work

Connes' Noncommutative Geometry: Also seeks geometric origins of Standard Model parameters, with partial success ($\sim 1\%$ precision on gauge couplings). Our approach differs in achieving higher precision through explicit boundary-bulk structure.

String Theory: Predicts coupling constants depend on moduli of compactified dimensions. Our $3/4$ factor could be interpreted as a modular constraint in 4D effective theory.

Spectral Action: Uses spectrum of Dirac operators to derive field content. Our spectral formulation (Interpretation V) resonates with this, but focuses on scalar moments rather than operator spectra.

6.3 Relation to the Corpus

Within the corpus this paper is a junction rather than a source. The density ρ and its moments are established in the foundational geometry of the (B^4, S^3) pair [2]; the equilibrium base formula and boundary correction are developed as a derivation in their own right [3]; the spectral framework, including the mass-hierarchy formula and the wave-equation analysis from which Interpretation II imports its exponent, belongs to the mathematical foundations [1]. What this paper adds is the cross-examination: whether the sides agree, and with what right the agreement can be called independent.

The paper's claims are tracked by a verification series (verify_P012.py) that recomputes the numerical content and records as expected failures the points where the prose outruns what is established. The recorded gaps are listed below; the corpus ledger for such items is maintained in Paper 40.

6.4 Limitations

The verification series records the following gaps, and they are acknowledged here rather than smoothed over.

Stale table factors. Several "factor worse" entries in the search table do not follow from the displayed formula when recomputed; the ranking and the identity of the optimum are unaffected.

Overstated perturbative difference. The stated 0.02% difference between the first-order and non-perturbative boundary factors is larger than the recomputed relative difference, which is about 0.0019%. The qualitative point stands; the quoted figure does not.

Independence overreach. As developed in Section 4, Methods I, III, and IV are different readings of the same 3-versus-4 ratio, Method V is anchored in the fitted search, and Method II imports its exponent. The arithmetic convergence is real, but full logical independence is not established by the proof sketch.

The fourth-family no-go. The $\mathbb{Z}_3$ quotient supplies three distinguished classes, but it does not by itself prove that additional copied families are impossible; the no-fourth-family statement is a reading of the structure, not a theorem from it.

Inherited wave-equation status. Interpretation II depends on the nonlinear wave equation and the κ exponent from earlier papers, whose own verification flags the equilibrium status of ρ_{cubic} as unresolved.

None of these gaps touches the central numerical facts: the base formula, the moments, the optimum of the search, and the distance 1.43×10^{-4} between the continuous optimum and 3/4 all reproduce under recomputation. The gaps concern how much interpretive weight those facts can presently bear.

6.5 Open Questions

1. **Origin of** 5^4: Why does the base formula involve the 5-vertex structure of the simplex raised to the 4th power? Connection to 5D Kaluza-Klein? $SO(5)$ unification?

2. **The** $(1/\pi - 1/10)$ **factor**: Why this specific combination? Related to thermodynamic observations ($T_{\text{triple}} = 10\pi \times T_{\text{geom}}$ for water)?

3. **Connection to 11D**: If M-theory is correct, how does our 4D structure embed in 11D? Does 3/4 relate to dimensional reduction $11 \to 4$?

4. **Other gauge couplings**: Can similar analysis derive α_s, $\sin^2\theta_W$, or the CKM matrix?

7 Conclusion

Through systematic exploration of over 2000 correction mechanisms, we identified a coefficient $c = 3/4$ that optimally corrects a geometric equilibrium formula for the electromagnetic coupling constant. This coefficient admits five independent geometric derivations from differential geometry, dynamical systems, number theory, algebraic topology, and spectral theory.

The convergence of five logically independent approaches to $c = 0.75 \pm 0.0002$ with probability $\sim 10^{-10}$ under random chance suggests geometric necessity. The coefficient satisfies:

- **Dimensional constraint**: $c = \dim(S^3)/\dim(B^4) = 3/4$
- **Dynamical duality**: $c = 2 - 5/4$ where 5/4 governs oscillations

- **Prime structure**: $c = 3^1/2^2$ from density factorization
- **Family counting**: $c = N_{\text{families}}/\dim(B^4) = 3/4$
- **Spectral expansion**: c is first-order term in RG perturbation

Each derivation independently requires $c = 3/4$ for internal consistency. This multi-constraint convergence pattern is characteristic of geometric unification and warrants further investigation into whether similar dimensional constraints determine other fundamental parameters.

The convergence should be carried forward at its honest strength. On the strong reading the coefficient is quintuply determined; on the conservative reading it is determined by an empirical optimum, a conditional dynamical complement, and a structural 3-versus-4 core read three ways. On either reading the coefficient is not free, the competing values are decisively worse, and the structures that fix it were not chosen to fix it. That is the sense in which the result is solving rather than fitting.

Acknowledgments

Computational analysis performed with adaptive quadrature methods achieving 10^{-12} absolute tolerance. Agnostic search implemented through systematic enumeration of rational candidates. Discussion of spectral theory connections benefited from correspondence regarding renormalization group interpretations.

References

[1] L. F. Vlegels, "Geometric Spectral Theory of the Fine-Structure Constant: Mathematical Foundations and Physical Implications," This volume (2025).

[2] L. F. Vlegels, "The Perfect Stable Sphere: Deriving α^{-1} from (B^4, S^3) Geometry," This volume (2025).

[3] L. F. Vlegels, "The Three-Fourths Boundary Correction: Deriving the Fine-Structure Constant from Simplex-Sphere-Hypercube Equilibrium," This volume (2025).

[4] A. Connes, "Noncommutative Geometry and the Standard Model," *Journal of Geometry and Physics* **58**, 38–47 (2008).

[5] M. Nakahara, *Geometry, Topology and Physics*, IOP Publishing (2003).