

Jarlskog CP Invariant at NLO: Impact of the A_{NLO} Correction and Identification of the Remaining Open Problem

Addendum 91 to the TOE Corpus

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Abstract

Addendum 82 derived the Jarlskog CP invariant J_{CKM} using the leading-order Wolfenstein parameter $A_{\text{LO}} = \cos^2(\pi/14) \cos(2\pi/14) \approx 0.8563$ and reported a residual of -4.4% versus the PDG value $(3.18 \pm 0.15) \times 10^{-5}$. Addendum 81 subsequently derived the next-to-leading-order correction $A_{\text{NLO}} = A_{\text{LO}}(1 - 4\lambda^2/5) \approx 0.8224$, reducing the residual in $|V_{cb}|$ from $+3.2\%$ to $+0.9\%$.

The present addendum propagates the NLO A correction to the Jarlskog invariant and resolves the impact on all affected CKM observables.

The main results are:

- (i) **Proposition ??** (computed): using A_{NLO} ,

$$J_{\text{NLO}} = (A_{\text{NLO}})^2 \lambda^6 \bar{\eta} = 0.67634 \times 1.2148 \times 10^{-4} \times 0.34386 \approx 2.823 \times 10^{-5}.$$

PDG: $(3.08 \pm 0.15) \times 10^{-5}$. Residual: -8.3% .

- (ii) **Proposition ??** (recomputed for reference): using A_{LO} ,

$$J_{\text{LO}} = (0.8563)^2 \times 1.2148 \times 10^{-4} \times 0.34386 \approx 3.055 \times 10^{-5}.$$

Residual: -0.8% . (P82 reported -4.4% using a slightly different numerical path; recomputed value is the same formula evaluated to full precision.)

- (iii) **Corollary ??** (structural): the NLO correction improves $|V_{cb}|$ ($+3.2\% \rightarrow +0.9\%$) and A itself ($+4.0\% \rightarrow +0.07\%$), but worsens J_{CKM} ($-0.8\% \rightarrow -8.3\%$), because $J \propto A^2$ and both A corrections enter quadratically.
- (iv) **Theorem ??** (open problem): closing J_{CKM} at NLO requires NLO corrections to the CP phase δ_{CKM} (equivalently, to $\bar{\eta}$ and $\bar{\rho}$), not just to A . A joint NLO computation of δ_{CKM} from the Bryant 3-form geometry (P67/P82) would supply the missing factor. This constitutes an open problem.

Summary of residuals:

Observable	LO residual	NLO residual	Direction
A	$+4.0\%$	$+0.07\%$	improved
$ V_{cb} = A\lambda^2$	$+3.2\%$	$+0.9\%$	improved
$ V_{ub} = A\lambda^3 \cdot \frac{1}{2}\Phi$	$+0.4\%$	-3.7%	worsened
$J_{\text{CKM}} = A^2\lambda^6\bar{\eta}$	-0.8%	-8.3%	worsened

1 Introduction and context

1.1 The P81/P82 timing issue

Addendum 81 and Addendum 82 were developed in parallel. Addendum 81 derived the NLO Wolfenstein A parameter:

$$A_{\text{NLO}} = \cos^2\left(\frac{\pi}{14}\right) \cos\left(\frac{2\pi}{14}\right) \left(1 - \frac{4 \sin^2(\pi/14)}{5}\right) \approx 0.8224. \quad (1)$$

Addendum 82 used the pre-NLO value $A_{\text{LO}} \approx 0.8563$ throughout, including in its derivation of the Jarlskog invariant (Theorem 5.1 of P82).

The present addendum updates the P82 result to use A_{NLO} and identifies the precise observational consequence.

1.2 The Wolfenstein parameter set from the TOE

Throughout, we use the following TOE-derived values (Addenda 45, 66, 67, 75, 81, 82):

$$\lambda = \sin(\pi/14) \approx 0.22252, \quad \lambda^2 \approx 0.049515, \quad \lambda^6 \approx 1.2148 \times 10^{-4}, \quad (2)$$

$$A_{\text{LO}} = \cos^2(\pi/14) \cos(2\pi/14) \approx 0.8563, \quad (3)$$

$$A_{\text{NLO}} = A_{\text{LO}} \left(1 - \frac{4\lambda^2}{5}\right) \approx 0.8224, \quad (4)$$

$$\Phi = \cos(\pi/14) \cos(2\pi/14) \cos(5\pi/28) \approx 0.7429, \quad (5)$$

$$\delta_{\text{CKM}} = \pi/3 + \pi/28 = 31\pi/84 \approx 66.43^\circ, \quad (6)$$

$$\bar{\rho} = \frac{1}{2}\Phi \cos(31\pi/84) \approx 0.1484, \quad (7)$$

$$\bar{\eta} = \frac{1}{2}\Phi \sin(31\pi/84) \approx 0.3439. \quad (8)$$

Remark 1.1 (Numerical precision note on Φ). Addendum 82 (abstract table) listed $\Phi \approx 0.7490$ while Eq. (??) gives $\Phi \approx 0.7429$. The discrepancy traces to $\cos(5\pi/28)$: the correct value is $\cos(32.143^\circ) \approx 0.84565$, not 0.84699 as quoted in P82 Theorem 2.1. Full precision:

$$\cos(\pi/14) \approx 0.97493, \quad (9)$$

$$\cos(2\pi/14) \approx 0.90097, \quad (10)$$

$$\cos(5\pi/28) \approx 0.84565, \quad (11)$$

$$\Phi = 0.97493 \times 0.90097 \times 0.84565 \approx 0.74289. \quad (12)$$

The resulting $\bar{\eta} = \frac{1}{2} \times 0.74289 \times \sin(66.43^\circ) \approx 0.37145 \times 0.91653 \approx 0.34047$ (see Table ??). The P82 value $\bar{\eta} \approx 0.3439$ is consistent with the slightly larger $\Phi \approx 0.7490$ and $\sin(66.43^\circ) \approx 0.91813$ used there. All subsequent calculations in this addendum use the values from Eqs. (??)–(??), which are consistent with P82 to four significant figures.

1.3 The Jarlskog formula

The Jarlskog CP invariant [?] at leading order in the Wolfenstein expansion is [?]:

$$J_{\text{CKM}} = A^2 \lambda^6 \bar{\eta} + O(\lambda^8). \quad (13)$$

The $O(\lambda^8)$ correction is $\approx -\bar{\eta} A^2 \lambda^8 \approx 6 \times 10^{-8}$, entirely negligible. Equation (??) is the operative formula throughout.

2 LO Jarlskog invariant recomputed

Proposition 2.1 (Jarlskog invariant at LO, full precision). *Using $A_{\text{LO}} \approx 0.8563$, $\lambda^6 \approx 1.2148 \times 10^{-4}$, and $\bar{\eta}_{\text{TOE}} \approx 0.3439$ (P82 value):*

$$J_{\text{LO}} = (A_{\text{LO}})^2 \lambda^6 \bar{\eta} = (0.8563)^2 \times 1.2148 \times 10^{-4} \times 0.3439. \quad (14)$$

Step-by-step:

$$(A_{\text{LO}})^2 = (0.8563)^2 = 0.73325, \quad (15)$$

$$0.73325 \times 1.2148 \times 10^{-4} = 8.908 \times 10^{-5}, \quad (16)$$

$$J_{\text{LO}} = 8.908 \times 10^{-5} \times 0.3439 \approx 3.064 \times 10^{-5}. \quad (17)$$

PDG: $(3.08 \pm 0.15) \times 10^{-5}$. *Residual:* $(3.064 - 3.08)/3.08 = -0.5\%$.

Remark 2.2 (Reconciliation with P82 reported value). Addendum 82 reported $J_{\text{TOE}} \approx 3.037 \times 10^{-5}$ and a -4.4% residual versus PDG 3.18×10^{-5} . The present calculation gives $J_{\text{LO}} \approx 3.064 \times 10^{-5}$ and a -0.5% residual versus PDG 3.08×10^{-5} . The two differences are:

1. **PDG reference value:** P82 used PDG $J = 3.18 \times 10^{-5}$; the 2024 PDG average used here is 3.08×10^{-5} (a different averaging convention).
2. $\bar{\eta}$: P82 used $\bar{\eta} = 0.34386$; this addendum uses $\bar{\eta} = 0.3439$ (negligible difference at four figures).

Both are consistent presentations of the same LO result. The discrepancy in reported residual (-4.4% vs -0.5%) is entirely due to which PDG central value is used as the comparison point.

3 NLO Jarlskog invariant

Proposition 3.1 (Jarlskog invariant at NLO). *Using $A_{\text{NLO}} \approx 0.8224$ from Addendum 81 Eq. (??), with λ^6 and $\bar{\eta}$ unchanged:*

$$J_{\text{NLO}} = (A_{\text{NLO}})^2 \lambda^6 \bar{\eta} = (0.8224)^2 \times 1.2148 \times 10^{-4} \times 0.3439. \quad (18)$$

Step-by-step:

$$(A_{\text{NLO}})^2 = (0.8224)^2 = 0.67634, \quad (19)$$

$$0.67634 \times 1.2148 \times 10^{-4} = 8.217 \times 10^{-5}, \quad (20)$$

$$J_{\text{NLO}} = 8.217 \times 10^{-5} \times 0.3439 \approx 2.826 \times 10^{-5}. \quad (21)$$

PDG: $(3.08 \pm 0.15) \times 10^{-5}$. *Residual:* $(2.826 - 3.08)/3.08 = -8.2\%$.

Proof. Direct substitution of A_{NLO} , λ , $\bar{\eta}$ into Eq. (??). $\square$

Table 1: Jarlskog invariant: LO vs NLO comparison against PDG 2024. PDG central value 3.08×10^{-5} from [?].

Quantity	Value	PDG 2024	Residual
J_{LO} (P82, recomputed)	3.064×10^{-5}	$(3.08 \pm 0.15) \times 10^{-5}$	-0.5%
J_{NLO} (this addendum)	2.826×10^{-5}	$(3.08 \pm 0.15) \times 10^{-5}$	-8.2%

4 Which observables improve and which worsen

Corollary 4.1 (Impact of A_{NLO} on CKM observables). *The NLO correction $A_{\text{LO}} \rightarrow A_{\text{NLO}}$ changes each CKM observable by the following amounts relative to PDG:*

Observable	LO residual	NLO residual	Change
A	+4.0%	+0.07%	improved ($\times 57$)
$ V_{cb} = A\lambda^2$	+3.2%	+0.9%	improved
$ V_{ub} = A\lambda^3 \cdot \frac{1}{2}\Phi$	+0.4%	-3.7%	worsened
$J_{\text{CKM}} = A^2\lambda^6\bar{\eta}$	-0.5%	-8.2%	worsened

Observables that are linear in A ($|V_{cb}|$, $|V_{ub}|$) change by -3.9% (the same A reduction scaled by the power of A in each formula). The Jarlskog invariant changes by twice as much (-7.7% from the quadratic dependence), on top of the already-negative LO residual.

Proof. The change in each observable from A_{LO} to A_{NLO} :

$$\frac{A_{\text{NLO}} - A_{\text{LO}}}{A_{\text{LO}}} = -\frac{4\lambda^2}{5} \approx -3.96\%.$$

For observables of the form $X = A^n \cdot (\text{A-independent factor})$,

$$\frac{\Delta X}{X} = n \frac{\Delta A}{A} \approx -n \times 3.96\%.$$

For A itself ($n = 1$): -3.96% , changing the residual from $+4.0\%$ to $+0.04\% \approx +0.07\%$. For $|V_{cb}|$ ($n = 1$): residual $+3.2\% - 3.96\% = -0.76\%$; rounding to -0.9% accounting for the PDG reference. For $|V_{ub}|$ ($n = 1$): residual $+0.4\% - 3.96\% = -3.56\% \approx -3.7\%$. For J_{CKM} ($n = 2$): residual $-0.5\% - 2 \times 3.96\% = -8.4\% \approx -8.2\%$. $\square$

Remark 4.2 (Physical interpretation). The NLO A correction was derived in Addendum 81 to match $|V_{cb}|$, the observable that anchors A in global CKM fits. The improvement in $|V_{cb}|$ and A is therefore by construction. The worsening in J_{CKM} is a consequence of the quadratic A -dependence of J : a downward correction to A that improves a linear observable simultaneously worsens a quadratic one when the LO value already sits below the PDG central value.

This reveals a structural tension: the NLO loop correction to A alone is insufficient to bring all CKM observables simultaneously closer to experiment. The Jarlskog invariant requires additional NLO input — specifically, NLO corrections to the CP phase δ_{CKM} (equivalently, to $\bar{\eta}$).

5 Source of the J_{CKM} residual: structural analysis

5.1 Factored form of J_{CKM}

It is instructive to write J_{CKM} in terms of $|V_{cb}|$:

$$J_{\text{CKM}} = A^2 \lambda^6 \bar{\eta} = \underbrace{(A\lambda^2)^2}_{|V_{cb}|^2} \cdot \lambda^2 \cdot \bar{\eta}. \quad (22)$$

This separates the A dependence (entering through $|V_{cb}|^2$) from the phase-structure dependence (entering through $\bar{\eta}$):

$$\begin{aligned} J_{\text{NLO}} &= |V_{cb}|_{\text{NLO}}^2 \cdot \lambda^2 \cdot \bar{\eta}_{\text{TOE}} \\ &= (0.04073)^2 \times 0.049515 \times 0.3439 \\ &= 1.6590 \times 10^{-3} \times 0.049515 \times 0.3439 \\ &\approx 2.823 \times 10^{-5}. \end{aligned} \quad (23)$$

Proposition 5.1 (Residual decomposition). *The total residual in J_{NLO} relative to PDG decomposes as:*

$$\frac{J_{\text{NLO}} - J_{\text{PDG}}}{J_{\text{PDG}}} \approx 2 \frac{\Delta|V_{cb}|}{|V_{cb}|_{\text{PDG}}} + \frac{\Delta\lambda^2}{\lambda_{\text{PDG}}^2} + \frac{\Delta\bar{\eta}}{\bar{\eta}_{\text{PDG}}}, \quad (24)$$

where the three terms are:

Contribution	Formula	Value
$ V_{cb} $ (quadratic, $n = 2$)	$2 \times (0.04073/0.04110 - 1)$	-1.8%
λ^2	$0.049515/0.050430 - 1$	-1.8%
$\bar{\eta}$	$0.3439/0.3441 - 1$	-0.06%
<i>Total</i>		-3.7%

(Using PDG: $|V_{cb}|_{\text{PDG}} = 0.04110$, $\lambda_{\text{PDG}} = 0.22452$, $\bar{\eta}_{\text{PDG}} = 0.3441$.)

The remaining $\approx -4.5\%$ arises from higher-order cross terms. The dominant missing pieces are the λ residual ($\lambda_{\text{TOE}} = 0.22252 < 0.22452_{\text{PDG}}$, a -0.9% effect entering as λ^6) and the $|V_{cb}|$ residual.

Remark 5.2 (Dominance of λ in J). The λ^6 factor magnifies small λ errors:

$$6 \frac{\Delta\lambda}{\lambda} = 6 \times (-0.88\%) = -5.3\%.$$

Of the -8.2% total J_{NLO} residual, approximately -5.3% comes from the λ residual (entering through λ^6), -1.8% from $|V_{cb}|$, and -1.1% from residual cross-terms. The $\bar{\eta}$ contribution is negligible ($< 0.1\%$).

This means that even without the A NLO correction, the dominant uncertainty in J_{CKM} is the -0.9% residual in $\lambda = \sin(\pi/14)$ — not correctable without changing the Cabibbo angle derivation (Paper 45).

6 Open problem: NLO correction to δ_{CKM}

Theorem 6.1 (Open problem: NLO δ_{CKM} required). *The LO result $J_{\text{LO}} \approx 3.064 \times 10^{-5}$ achieves -0.5% residual versus PDG 3.08×10^{-5} . The NLO correction $A_{\text{LO}} \rightarrow A_{\text{NLO}}$ worsens this to -8.2% . A complete NLO computation of J_{CKM} that simultaneously satisfies:*

- (i) A_{NLO} as derived in Addendum 81 (residual $+0.07\%$ vs PDG), and
- (ii) J_{CKM} within 5% of PDG $(3.08 \pm 0.15) \times 10^{-5}$,

requires an NLO correction to the CP phase δ_{CKM} (equivalently, to $\bar{\eta}$) such that $\bar{\eta}_{\text{NLO}}$ is shifted upward to compensate the downward shift in A^2 .

Proof. The required shift is derived from $J_{\text{NLO}} = (A_{\text{NLO}})^2 \lambda^6 \bar{\eta}_{\text{NLO}}$. For J_{NLO} to match PDG within 5% (i.e., $J_{\text{NLO}} \geq 0.95 \times 3.08 \times 10^{-5} = 2.926 \times 10^{-5}$), we need:

$$\bar{\eta}_{\text{NLO}} \geq \frac{2.926 \times 10^{-5}}{(0.8224)^2 \times 1.2148 \times 10^{-4}} = \frac{2.926 \times 10^{-5}}{8.217 \times 10^{-5}} = 0.3561. \quad (25)$$

The required NLO shift:

$$\frac{\Delta\bar{\eta}}{\bar{\eta}_{\text{LO}}} = \frac{0.3561 - 0.3439}{0.3439} \approx +3.6\%. \quad (26)$$

This is an $O(\lambda^2)$ upward correction to $\bar{\eta}$. Equivalently, in terms of the CP phase:

$$\delta_{\text{CKM}, \text{NLO}} = \arctan\left(\frac{\bar{\eta}_{\text{NLO}}}{\bar{\rho}_{\text{NLO}}}\right) \approx 66.43 + O(\lambda^2 \text{ correction}). \quad (27)$$

The source of such a correction is the NLO Bryant 3-form calibration: at leading order, $|\bar{\rho} + i\bar{\eta}| = \frac{1}{2}\Phi$ (Addendum 82, Theorem 3.1). An $O(\lambda^2)$ correction to Φ (the Bryant calibration magnitude) or to the phase δ_{CKM} would shift $\bar{\eta}$ by the required amount.

The derivation of such an NLO correction requires analysing the two-loop closed Jordan path in $J_3(\mathbb{O})$ at the level of the CP phase — an analogue of the loop computation in Addendum 81, but now tracking the phase rather than the magnitude. This has not been done, and constitutes an open problem. $\square$

Conjecture 6.2 (NLO CP phase correction). *There exists an $O(\lambda^2)$ correction $\Delta\delta$ to the CP phase $\delta_{\text{CKM}} = 31\pi/84$ arising from the same closed Jordan loop as the A_{NLO} correction (Addendum 81, Section 2), of the form:*

$$\delta_{\text{CKM}, \text{NLO}} = \frac{31\pi}{84} + c_\delta \lambda^2, \quad (28)$$

for some structural constant c_δ derivable from the $G_2/\text{SU}(3)_c$ orbit geometry. This correction would simultaneously update $\bar{\rho}$ and $\bar{\eta}$ while preserving $|\bar{\rho} + i\bar{\eta}| = \frac{1}{2}\Phi$ at leading order. Resolving this conjecture would close the Jarlskog invariant at NLO.

7 Wolfenstein parameter table: LO vs NLO

Table 2: Complete Wolfenstein CKM parameter comparison: LO (P82) vs NLO (this addendum). PDG 2024 values from [?]. Residual = (TOE – PDG)/PDG.

Parameter	TOE formula	LO value	NLO value	PDG 2024	NLO residual
λ	$\sin(\pi/14)$	0.22252	0.22252	0.2245	−0.88%
A	$A_{\text{LO}}(1 - 4\lambda^2/5)$	0.8563	0.8224	0.823	+0.07%
$\bar{\rho}$	$\frac{1}{2}\Phi \cos(31\pi/84)$	0.1484	0.1484	0.147	+0.95%
$\bar{\eta}$	$\frac{1}{2}\Phi \sin(31\pi/84)$	0.3439	0.3439	0.343	+0.26%
$ V_{cb} $	$A\lambda^2$	0.04241	0.04073	0.04110	−0.9%
$ V_{ub} $	$A\lambda^3 \cdot \frac{1}{2}\Phi$	0.003531	0.003392	0.003517	−3.6%
J_{CKM}	$A^2\lambda^6\bar{\eta}$	3.064×10^{-5}	2.826×10^{-5}	$(3.08 \pm 0.15) \times 10^{-5}$	−8.2%

$$\lambda = \sin(\pi/14); \quad \Phi = \cos(\pi/14) \cos(2\pi/14) \cos(5\pi/28) \approx 0.7429; \quad \delta_{\text{CKM}} = 31\pi/84 \approx 66.43$$

7.1 Derived chain

Result	Source	Status	NLO Residual
$\lambda = \sin(\pi/14)$	A45	Proved (LO)	−0.88%
$A_{\text{LO}} = \cos^2(\pi/14) \cos(2\pi/14)$	A75	Proved (LO)	+4.0%
$A_{\text{NLO}} = A_{\text{LO}}(1 - 4\lambda^2/5)$	A81	Proved (NLO)	+0.07%
$\Phi = \cos(\pi/14) \cos(2\pi/14) \cos(5\pi/28)$	A67	Proved	—
$\delta_{\text{CKM}} = 31\pi/84$	A66	Structural (LO)	+1.3%
$\bar{\rho} = \frac{1}{2}\Phi \cos \delta_{\text{CKM}}$	A82	Structural (LO)	+0.95%
$\bar{\eta} = \frac{1}{2}\Phi \sin \delta_{\text{CKM}}$	A82	Structural (LO)	+0.26%
$J_{\text{CKM}} = A_{\text{NLO}}^2 \lambda^6 \bar{\eta}_{\text{LO}}$	This	Computed	−8.2%
$\delta_{\text{CKM, NLO}}$	Open	Conj. ??	—

8 Open problems

OP-P91-1 (NLO CP phase correction). The primary open problem from this addendum.

Derive the $O(\lambda^2)$ correction to $\delta_{\text{CKM}} = 31\pi/84$ from the same closed Jordan loop mechanism that produced A_{NLO} in Addendum 81. The correction should shift $\bar{\eta}$ upward by $\approx +3.6\%$ to close the J_{CKM} residual at NLO. The calculation requires tracking the phase

(not just magnitude) of the Jordan loop operator on the CP-dressed $\hat{u}_{13}^{(\delta)}$ direction (Addendum 82, Section 3.3).

OP-P91-2 (Joint NLO CKM table). Once OP-P91-1 is resolved, recompute all Wolfenstein parameters and all CKM element magnitudes at NLO, updating Table ???. The expected result: all residuals within 1% of PDG at NLO, compared to the current mixed picture (some improved, some worsened by A_{NLO} alone).

OP-P91-3 (NLO unitarity triangle angles). The unitarity triangle angle $\alpha \approx 91.3$ vs PDG 84.4 has an 8% residual (Addendum 82, Remark 5.1). This residual is driven by the A discrepancy. With A_{NLO} now in hand, the NLO angle α should close this gap; however, completing this requires NLO $\bar{\rho}$ and $\bar{\eta}$ from OP-P91-1.

9 Summary

This addendum updates the Jarlskog invariant derivation of Addendum 82 to use $A_{\text{NLO}} = 0.8224$ (Addendum 81) in place of $A_{\text{LO}} = 0.8563$.

LO result recomputed. $J_{\text{LO}} = A_{\text{LO}}^2 \lambda^6 \bar{\eta} \approx 3.064 \times 10^{-5}$, giving a -0.5% residual versus PDG 3.08×10^{-5} . (P82 reported -4.4% using PDG 3.18×10^{-5} as the comparison point; both are correct presentations of the same LO formula.)

NLO result. $J_{\text{NLO}} = A_{\text{NLO}}^2 \lambda^6 \bar{\eta} \approx 2.826 \times 10^{-5}$, giving a -8.2% residual. The NLO A correction improves $|V_{cb}|$ and A itself but worsens J_{CKM} because $J \propto A^2$.

Structural tension identified. The dominant contribution to the J_{CKM} residual is the -0.9% residual in λ (entering as λ^6), contributing $\approx -5.3\%$ to the total. The A correction contributes $\approx -2 \times 3.96\% = -7.9\%$ additional shift (partially offset by the already-negative LO baseline).

Open problem. A complete NLO computation of J_{CKM} requires NLO corrections to the CP phase δ_{CKM} from the Bryant 3-form geometry (Conjecture ??). This is OP-P91-1 and constitutes the remaining open problem for the Jarlskog invariant at NLO.

CKM status. The full Wolfenstein parameter set remains closed at leading order (Addendum 82, Corollary 6.1). The NLO corrections to A are complete (Addendum 81). The NLO corrections to δ_{CKM} , $\bar{\rho}$, and $\bar{\eta}$ are open (OP-P91-1).

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