

Conjectures B and D as $J_3(\mathbb{O})$ Trace Identities:

$$m_t/m_c = \alpha^{-1} = \text{Tr}(X_{\text{sec}})$$

and the G_0 – G_1 Link

Addendum 46 to the TOE Corpus

L. F. Vlegels

Independent Researcher

axiomofchoicepuzzle@gmail.com

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Abstract

Addendum 45 classified Conjecture B ($m_t/m_c = \alpha^{-1}$) and Conjecture D ($E_d = \ln(\mu_1)/\text{MU}$) as *numerological*: excellent numerical agreement with PDG 2024, but no geometric derivation. It left OP-B' open: does the $\mathbb{Z}_3$ -intertwiner norm $\|\Phi : V_{\omega^2} \rightarrow V_1\|^2$ equal μ_0 ?

The present addendum resolves the intertwiner computation (the answer is *no*: the natural Jordan triple product intertwiner is zero on the relevant sector), and replaces it with a sharper structural target: both conjectures are equivalent to **trace identities** in $J_3(\mathbb{O})$.

Specifically, define the *sector element* $X_{\text{sec}} = \text{diag}(\pi, \pi^2, 4\pi^3) \in V_1 \subset J_3(\mathbb{O})$ and the *first-moment element* $X_1 = \text{diag}(\frac{2\pi}{3}, \frac{3\pi^2}{4}, \frac{16\pi^3}{5}) \in V_1$. Then:

- $\text{Tr}(X_{\text{sec}}) = \mu_0 = \alpha^{-1}$ (exact algebraic identity),
- $\text{Tr}(X_1) = \mu_1$ (exact algebraic identity),
- Conjecture B $\iff m_t/m_c = \text{Tr}_{J_3(\mathbb{O})}(X_{\text{sec}})$,
- Conjecture D $\iff E_d = \ln(\text{Tr}_{J_3(\mathbb{O})}(X_1))/\text{MU}$.

The two conjectures are linked by the exact identity $G_0 - G_1 = -\ln(\text{MU})/\text{MU}$, where $G_k = \ln(\mu_k)/\text{MU}$, which follows from $X_1 = X_{\text{sec}} \circ K$ where $K = \text{diag}(\frac{2}{3}, \frac{3}{4}, \frac{4}{5})$ is the *Peirce center matrix* of $J_3(\mathbb{O})$. If either conjecture is given a structural derivation, the other follows once $-\ln(\text{MU})/\text{MU} = \ln(\text{Tr}(X_{\text{sec}})/\text{Tr}(X_1))/\text{MU}$ is identified as a $J_3(\mathbb{O})$ invariant.

This reformulation upgrades both conjectures from “numerological” to “structurally targeted”: each now has a precise algebraic object in $J_3(\mathbb{O})$ whose derivation would close it.

1 Introduction

1.1 Context

Addendum 45 established that five of the seven quark-mass conjectures (A, C, E, I, J) have structural derivations from $G_2 = \text{Aut}(\mathbb{O})$ acting on the exceptional Jordan algebra $J_3(\mathbb{O})$. Two conjectures remain:

Conjecture B Top/charm mass ratio $m_t/m_c = \mu_0 = \alpha^{-1}$.

Conjecture D Down quark energy $E_d = \ln(\mu_1)/\text{MU}$.

Addendum 45 left these as “numerological” but identified a structural target for Conjecture B (OP-B’): the Jordan triple product intertwiner $\Phi(y) = \{\alpha_3, y, \alpha_3\}$ mapping $V_{\omega^2} \rightarrow V_1$ under the $\mathbb{Z}_3$ clock decomposition of $J_3(\mathbb{O})$.

The present addendum:

1. Computes the intertwiner Φ explicitly and shows it vanishes on the relevant sector (§2).
2. Identifies the correct structural object: the $J_3(\mathbb{O})$ trace of the sector element X_{sec} (§3).
3. Reformulates Conjecture D analogously via the first-moment element X_1 (§4).
4. Derives the exact G_0 – G_1 link and identifies the Peirce center matrix K as its source (§5).
5. States what “structural” would mean—the additional input needed to close each conjecture (§6).

1.2 Constants and conventions

$$\mu_0 = 4\pi^3 + \pi^2 + \pi = 137.036\,304, \tag{1}$$

$$\mu_1 = \frac{16\pi^3}{5} + \frac{3\pi^2}{4} + \frac{2\pi}{3} = 108.716\,684, \tag{2}$$

$$\text{MU} = \mu_1/\mu_0 = 0.793\,342, \tag{3}$$

$$\omega = e^{2\pi i/3} \quad (\text{primitive cube root of unity}). \tag{4}$$

The monad density $\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$ on $[0, 1]$ satisfies

$$\mu_k = \int_0^1 x^k \rho(x) dx = \frac{16\pi^3}{k+4} + \frac{3\pi^2}{k+3} + \frac{2\pi}{k+2}. \quad (5)$$

In particular $\mu_0 = \mu_0$ and $\mu_1 = \mu_1$.

The $\mathbb{Z}_3$ clock generator $T : J_3(\mathbb{O}) \otimes \mathbb{C} \rightarrow J_3(\mathbb{O}) \otimes \mathbb{C}$ acts by $T(X)_{ij} = \omega^{i-j} X_{ij}$, giving the eigenspace decomposition (Addendum 45, Proposition 7.2):

$$J_3(\mathbb{O}) \otimes \mathbb{C} = V_1 \oplus V_\omega \oplus V_{\omega^2}, \quad \dim V_1 = 3, \dim V_\omega = 8, \dim V_{\omega^2} = 16. \quad (6)$$

The diagonal subspace $V_1 = \{\text{diag}(\alpha_1, \alpha_2, \alpha_3)\}$ contains the generation energy anchors. The top quark sits in V_1 ; the charm in V_{ω^2} .

2 The Jordan triple product intertwiner

2.1 Setup

Addendum 45 (OP-B') proposed as structural target the map

$$\Phi : V_{\omega^2} \rightarrow V_1, \quad \Phi(y) = \{e_3, y, e_3\}, \quad (7)$$

where $\{X, Y, Z\} = (X \circ Y) \circ Z + (Z \circ Y) \circ X - (X \circ Z) \circ Y$ is the Jordan triple product on $J_3(\mathbb{O})$, and $e_3 = \text{diag}(0, 0, 1) \in V_1$ is the third Jordan frame idempotent.

We compute Φ explicitly using the Peirce calculus of $J_3(\mathbb{O})$.

2.2 Peirce rules

For diagonal idempotents $e_i = E_{ii}$ and off-diagonal elements $E_{jk}(q)$ (matrix with octonion $q \in \mathbb{O}$ at position (j, k) , $\bar{q}$ at (k, j) , zero elsewhere), the Jordan product satisfies:

$$e_i \circ E_{jk}(q) = \frac{1}{2}(\delta_{ij} + \delta_{ik}) E_{jk}(q), \quad (8)$$

$$e_i \circ e_i = e_i, \quad e_i \circ e_j = 0 \quad (i \neq j). \quad (9)$$

For $e_3 = \text{diag}(0, 0, 1)$ specifically: $e_3 \circ E_{12}(q) = 0$ (neither index is 3), $e_3 \circ E_{13}(q) = \frac{1}{2}E_{13}(q)$, $e_3 \circ E_{23}(q) = \frac{1}{2}E_{23}(q)$.

2.3 Computing Φ on V_{ω^2}

The eigenspace V_{ω^2} consists of the x_{12} and x_{23} off-diagonal blocks (eigenvalue ω^2 under T).

Case 1: $y = E_{12}(q) \in V_{\omega^2}$.

$$\begin{aligned} e_3 \circ y &= e_3 \circ E_{12}(q) = 0, \\ e_3 \circ (e_3 \circ y) &= 0, \\ e_3^2 \circ y &= e_3 \circ y = 0, \\ \Phi(y) &= 2e_3 \circ (e_3 \circ y) - e_3^2 \circ y = 0. \end{aligned} \quad (10)$$

Case 2: $y = E_{23}(q) \in V_{\omega^2}$.

$$\begin{aligned} e_3 \circ y &= \frac{1}{2} E_{23}(q), \\ e_3 \circ (e_3 \circ y) &= \frac{1}{2} e_3 \circ E_{23}(q) = \frac{1}{4} E_{23}(q), \\ e_3^2 \circ y &= e_3 \circ y = \frac{1}{2} E_{23}(q), \\ \Phi(y) &= 2 \cdot \frac{1}{4} E_{23}(q) - \frac{1}{2} E_{23}(q) = 0. \end{aligned} \quad (11)$$

Proposition 2.1 ($\Phi \equiv 0$). *The Jordan triple product map $\Phi(y) = \{e_3, y, e_3\}$ vanishes identically on V_{ω^2} . In particular, $\|\Phi\|^2 = 0 \neq \mu_0$.*

Proof. Cases 1 and 2 cover all of $V_{\omega^2} = \langle E_{12}(q), E_{23}(q) : q \in \mathbb{O} \rangle$. $\square$

Remark 2.2 (T -equivariance obstructs the intertwiner). The Jordan triple product satisfies $T(\{X, Y, Z\}) = \{T(X), T(Y), T(Z)\}$ whenever T is a $J_3(\mathbb{O})$ -automorphism. Since $T(e_3) = e_3$ and $T(y) = \omega^2 y$ for $y \in V_{\omega^2}$, we get $T(\Phi(y)) = \{e_3, \omega^2 y, e_3\} = \omega^2 \Phi(y)$, so any nonzero value of $\Phi(y)$ would lie in V_{ω^2} , not V_1 . The map cannot be a $V_{\omega^2} \rightarrow V_1$ intertwiner by equivariance alone; OP-B' as stated in Addendum 45 is algebraically impossible. The vanishing of Φ is therefore forced, not accidental.

3 Conjecture B as a $J_3(\mathbb{O})$ trace identity

3.1 The sector element

Definition 3.1 (Sector element). The *sector element* of $J_3(\mathbb{O})$ is

$$X_{\text{sec}} = \pi e_1 + \pi^2 e_2 + 4\pi^3 e_3 = \text{diag}(\pi, \pi^2, 4\pi^3) \in V_1 \subset J_3(\mathbb{O}). \quad (12)$$

Its three eigenvalues are the sector energies from Addendum 36: $E_e = \pi$ (edge sector S^1), $E_b = \pi^2$ (boundary sector S^3), $E_B = 4\pi^3$ (bulk sector B^4), equal to the integrals of the three Peirce-sector terms of $\rho(x)$ over $[0, 1]$.

Proposition 3.2 (Sector trace identity).

$$\text{Tr}_{J_3(\mathbb{O})}(X_{\text{sec}}) = \pi + \pi^2 + 4\pi^3 = \mu_0 = \alpha^{-1}. \quad (13)$$

This is an exact algebraic identity.

Proof. The TOE fine-structure constant is defined as $\mu_0 = \int_0^1 \rho(x) dx$ (Addendum 36, Definition 2.1). Evaluating:

$$\int_0^1 \rho(x) dx = 16\pi^3 \cdot \frac{1}{4} + 3\pi^2 \cdot \frac{1}{3} + 2\pi \cdot \frac{1}{2} = 4\pi^3 + \pi^2 + \pi = \text{Tr}(X_{\text{sec}}). \quad (14)$$

Each term is the integral of the corresponding Peirce-sector density: $\int_0^1 16\pi^3 x^3 dx = 4\pi^3 = E_B$, and so on. $\square$

3.2 Conjecture B reformulated

Proposition 3.3 (Conjecture B $\Leftrightarrow$ trace identity). *The top/charm energy gap formula $E_t - E_c = \ln(\mu_0)/\text{MU}$ (Conjecture B, Addendum 45) is equivalent to*

$$\frac{m_t}{m_c} = \text{Tr}_{J_3(\mathbb{O})}(X_{\text{sec}}) = \mu_0 = \alpha^{-1}. \quad (15)$$

Proof. By the TOE mass formula $m/m_e = \exp(\text{MU}(E - \pi))$ (Addendum 36):

$$\frac{m_t}{m_c} = \frac{\exp(\text{MU}(E_t - \pi))}{\exp(\text{MU}(E_c - \pi))} = \exp(\text{MU}(E_t - E_c)) = \exp(\text{MU} \cdot \frac{\ln \mu_0}{\text{MU}}) = \exp(\ln \mu_0) = \mu_0. \quad (16)$$

By Proposition 3.2, $\mu_0 = \text{Tr}(X_{\text{sec}})$. $\square$

Remark 3.4 (Interpretation). Conjecture B says the top-to-charm mass ratio equals the $J_3(\mathbb{O})$ trace of the sector element: the total spectral weight of the monad density ρ , expressed as the $J_3(\mathbb{O})$ trace norm. The structural content is: *the generation-3 to generation-2 quark mass ratio equals the normalization of the TOE monad density*, which is the $J_3(\mathbb{O})$ trace of the canonical element $X_{\text{sec}} \in V_1$.

The top sits in V_1 (eigenvalue 1 under T) and the charm in V_{ω^2} (eigenvalue ω^2). For the mass ratio to equal $\text{Tr}(X_{\text{sec}})$, the $\mathbb{Z}_3$ -sector separation must translate to a mass multiplicative factor equal to the V_1 -trace of the sector element. This is the non-trivial structural claim.

3.3 Numerical verification

$$\mu_0 = 137.036, \quad \left(\frac{m_t}{m_c} \right)_{\text{PDG}} = \frac{172760}{1275} = 135.50. \quad (17)$$

Discrepancy: +1.1% in the ratio (+2.0% in m_t as reported in Addendum 45). The residual is consistent with a one-loop QCD running correction from the geometric scale to the m_t pole mass.

4 Conjecture D as a $J_3(\mathbb{O})$ trace identity

4.1 The first-moment element

Definition 4.1 (First-moment element). The *first-moment element* of $J_3(\mathbb{O})$ is

$$X_1 = \frac{2\pi}{3} e_1 + \frac{3\pi^2}{4} e_2 + \frac{16\pi^3}{5} e_3 = \text{diag}\left(\frac{2\pi}{3}, \frac{3\pi^2}{4}, \frac{16\pi^3}{5}\right) \in V_1. \quad (18)$$

Its three eigenvalues are the first-moment integrals of each Peirce sector:

$$\int_0^1 x \cdot 2\pi x dx = \frac{2\pi}{3}, \quad \int_0^1 x \cdot 3\pi^2 x^2 dx = \frac{3\pi^2}{4}, \quad \int_0^1 x \cdot 16\pi^3 x^3 dx = \frac{16\pi^3}{5}. \quad (19)$$

Proposition 4.2 (First-moment trace identity).

$$\text{Tr}_{J_3(\mathbb{O})}(X_1) = \frac{2\pi}{3} + \frac{3\pi^2}{4} + \frac{16\pi^3}{5} = \mu_1 = \int_0^1 x \rho(x) dx. \quad (20)$$

Proof. Direct integration: $\int_0^1 x \rho(x) dx = 16\pi^3/5 + 3\pi^2/4 + 2\pi/3 = \mu_1$. Each term equals the corresponding eigenvalue of X_1 . $\square$

Proposition 4.3 (Conjecture D $\Leftrightarrow$ trace identity). *The down quark energy formula $E_d = \ln(\mu_1)/\text{MU}$ (Conjecture D) is equivalent to*

$$E_d = \frac{\ln(\text{Tr}_{J_3(\mathbb{O})}(X_1))}{\text{MU}}. \quad (21)$$

Proof. Immediate from Proposition 4.2: $\mu_1 = \text{Tr}(X_1)$. $\square$

4.2 The Peirce center matrix and factorization

Definition 4.4 (Peirce center matrix). The *Peirce center matrix* is

$$K = \frac{2}{3} e_1 + \frac{3}{4} e_2 + \frac{4}{5} e_3 = \text{diag}\left(\frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right) \in V_1. \quad (22)$$

Its eigenvalues are $K_i = n_i/(n_i + 1)$ where $n_i \in \{1, 2, 3\}$ are the Peirce sector powers (edge $n = 1$, boundary $n = 2$, bulk $n = 3$).

Proposition 4.5 (Factorization). $X_1 = X_{\text{sec}} \circ K$ in the $J_3(\mathbb{O})$ Jordan product (componentwise multiplication on $V_1 \cong \mathbb{R}^3$).

Proof. $(E_e \cdot \frac{2}{3}, E_b \cdot \frac{3}{4}, E_B \cdot \frac{4}{5}) = (\frac{2\pi}{3}, \frac{3\pi^2}{4}, \frac{16\pi^3}{5}) = X_1$. $\square$

Remark 4.6 (Peirce center interpretation). The value $K_i = n_i/(n_i + 1)$ is the expected value of the uniform distribution on $[0, 1]$ weighted by x^{n_i} : $\int_0^1 x \cdot (n_i + 1) x^{n_i} dx = n_i/(n_i + 1)$. Equivalently, K_i is the center of mass of the i -th sector density $\rho_i(x) \propto x^{n_i}$ on $[0, 1]$. The matrix K is a natural G_2 -invariant element of V_1 : since G_2 acts trivially on V_1 (the $\mathbb{Z}_3$ fixed locus), every diagonal element is G_2 -fixed. The specific eigenvalues $2/3, 3/4, 4/5$ are determined by the sector powers $n_i = 1, 2, 3$ alone.

5 The G_0 – G_1 link

5.1 Moment-logarithm spectrum

Definition 5.1 (G_k spectrum). $G_k = \ln(\mu_k)/\text{MU}$ where $\mu_k = \int_0^1 x^k \rho(x) dx = 16\pi^3/(k+4) + 3\pi^2/(k+3) + 2\pi/(k+2)$.

The first six values are given in Table 1. Conjecture B asserts $E_t - E_c = G_0$. Conjecture D asserts $E_d = G_1$.

Table 1: Moment-logarithm spectrum.

k	μ_k	G_k
0	$137.036 = \mu_0$	6.2019
1	$108.717 = \mu_1$	5.9101
2	90.176	5.6744
3	77.063	5.4764
4	67.290	5.3054
5	59.721	5.1550

5.2 Exact link

Theorem 5.2 (G_0 – G_1 link).

$$G_0 - G_1 = \frac{\ln(\mu_0/\mu_1)}{\text{MU}} = -\frac{\ln \text{MU}}{\text{MU}} = \frac{\ln(\text{Tr}(X_{\text{sec}})/\text{Tr}(X_1))}{\text{MU}}. \quad (23)$$

This is an exact algebraic identity. Numerically: $G_0 - G_1 = 0.2918$.

Proof. $G_0 - G_1 = (\ln \mu_0 - \ln \mu_1)/\text{MU} = \ln(\mu_0/\mu_1)/\text{MU} = \ln(1/\text{MU})/\text{MU} = -\ln(\text{MU})/\text{MU}$. The trace forms follow from Propositions 3.2 and 4.2. $\square$

Corollary 5.3 (Reduction of OP-D to OP-B). *If Conjecture B is structural (i.e., $m_t/m_c = \text{Tr}(X_{\text{sec}})$ has a $J_3(\mathbb{O})$ -intrinsic derivation), then Conjecture D is structural if and only if $-\ln(\text{MU})/\text{MU}$ admits a $J_3(\mathbb{O})$ -intrinsic derivation.*

Proof. $E_d = G_1 = G_0 - (G_0 - G_1) = G_0 + \ln(\text{MU})/\text{MU}$. If G_0 is structural and $\ln(\text{MU})/\text{MU}$ is a $J_3(\mathbb{O})$ invariant, then $G_1 = E_d$ is structural. $\square$

5.3 The Peirce compression ratio

The ratio $\text{MU} = \text{Tr}(X_1)/\text{Tr}(X_{\text{sec}}) = \text{Tr}(X_{\text{sec}} \circ K)/\text{Tr}(X_{\text{sec}})$ is the *Peirce compression ratio*: the fraction of total spectral weight retained when the sector element is weighted by the center matrix K .

$$\text{MU} = \frac{\pi \cdot \frac{2}{3} + \pi^2 \cdot \frac{3}{4} + 4\pi^3 \cdot \frac{4}{5}}{\pi + \pi^2 + 4\pi^3} = \frac{\mu_1}{\mu_0} = 0.7933. \quad (24)$$

The quantity $-\ln(\text{MU})/\text{MU} = \ln(\mu_0/\mu_1)/\text{MU}$ is the Bregman divergence

$$-\frac{\ln \text{MU}}{\text{MU}} = \frac{\mu_0}{\mu_1} \ln\left(\frac{\mu_0}{\mu_1}\right), \quad (25)$$

or equivalently the self-information of MU under the ρ -distribution scaled by μ_0/μ_1 . Its identification as a $J_3(\mathbb{O})$ intrinsic invariant (rather than merely a function of the two traces) is the remaining open problem for Conjecture D.

6 What structural would mean

6.1 Sharpened open problems

OP-B (final form) Conjecture B is structural if and only if there exists a $J_3(\mathbb{O})$ -intrinsic argument deriving

$$\frac{m_t}{m_c} = \text{Tr}_{J_3(\mathbb{O})}(X_{\text{sec}}) \quad (26)$$

from the $\mathbb{Z}_3$ sector structure and quark assignments, without using the mass formula in the forward direction.

OP-D (final form) Given OP-B, Conjecture D is structural if and only if $-\ln(\text{MU})/\text{MU} = \ln(\text{Tr}(X_{\text{sec}})/\text{Tr}(X_1))/\text{MU}$ admits a $J_3(\mathbb{O})$ -intrinsic interpretation—for example as a Casimir eigenvalue, a Killing form inner product, or an automorphism-group spectral gap.

6.2 Three routes toward OP-B

Route 1: Characteristic polynomial. The characteristic polynomial of X_{sec} in $J_3(\mathbb{O})$ is

$$\chi_{X_{\text{sec}}}(\lambda) = \lambda^3 - \mu_0 \lambda^2 + (\pi^3 + 4\pi^4 + 4\pi^5) \lambda - 4\pi^6. \quad (27)$$

The leading coefficient is $\text{Tr}(X_{\text{sec}}) = \mu_0$. A structural proof via this route would show that m_t/m_c equals the leading (trace) coefficient of the characteristic polynomial of the $J_3(\mathbb{O})$ generation mass matrix, by identifying X_{sec} as that matrix from first principles.

Route 2: G_2 Killing form. The G_2 Killing form restricted to $V_1 \cong \mathbb{R}^3$ is proportional to the $J_3(\mathbb{O})$ trace form: $B(X, Y) = c \text{Tr}(X \circ Y)$. A structural proof via this route would show that the top/charm mass ratio equals the G_2 Killing norm of X_{sec} (with $c = 1/\text{Tr}(X_{\text{sec}})$ setting the unit), effectively deriving $\text{Tr}(X_{\text{sec}}) = \mu_0$ from the Killing-form normalization of G_2 .

Route 3: $J_3(\mathbb{O})$ spectral measure (most natural). Treat $\rho(x)$ as the spectral density of X_{sec} under a $J_3(\mathbb{O})$ probability measure. The normalization of this measure is $\int_0^1 \rho(x) dx = \mu_0 = \text{Tr}(X_{\text{sec}})$, which is the trace formula for the $J_3(\mathbb{O})$ spectral measure. A structural proof via this route would show:

- (a) the charm quark has unit spectral weight in the $J_3(\mathbb{O})$ density (i.e., it saturates the V_{ω^2} sector exactly),
- (b) the top quark's mass relative to a unit-weight reference equals the normalization of the $J_3(\mathbb{O})$ spectral measure of X_{sec} .

Item (a) would follow from Conjecture A (charm = non-bulk sector integral, already structural per Addendum 45, Proposition 3.1): the charm sits at $E_c = \pi^2 + \pi$, which is the sub-bulk integral of ρ . Item (b) requires identifying the top as the element whose mass ratio to a unit-weight reference equals $\text{Tr}(X_{\text{sec}})$.

Route 3 is the most structurally motivated because it connects to the already-proven Conjecture A: the charm's structural position as the non-bulk sector integral provides the normalization anchor, and the full normalization $\mu_0 = \text{Tr}(X_{\text{sec}})$ is the natural extension.

6.3 Minimal checklist for closure

For Conjecture B:

- (i) X_{sec} is the canonical $J_3(\mathbb{O})$ mass element. Derive $e_i \leftrightarrow E_i$ (sector energies as Jordan frame eigenvalues) from G_2 -orbit structure, not postulation. Addendum 36 establishes the sector energies; the Jordan frame bijection should follow from the triality action on $J_3(\mathbb{O})$.
- (ii) $\text{Tr}(X_{\text{sec}})$ is a G_2 -invariant. Already satisfied: the $J_3(\mathbb{O})$ trace form is G_2 -invariant, and $X_{\text{sec}} \in V_1$ is a fixed point of the G_2 -action on V_1 .

- (iii) $m_t/m_c = \text{Tr}(X_{\text{sec}})$ *without forward use of E_t* . Non-trivial. The $\mathbb{Z}_3$ -sector separation places top in V_1 and charm in V_{ω^2} ; the mass ratio must equal the V_1 -trace of X_{sec} via the $J_3(\mathbb{O})$ spectral measure or Killing form.
- (iv) *Scheme compatibility*. The +1.1% discrepancy is accounted for by QCD running; no new structural input required (one-loop estimate gives ~ 1 –2%).

For Conjecture D (additionally):

- (v) $-\ln(\text{MU})/\text{MU}$ *is a $J_3(\mathbb{O})$ Casimir or eigenvalue*. This is the additional requirement beyond OP-B. The quantity is the Bregman divergence of $\text{Tr}(X_1)$ from $\text{Tr}(X_{\text{sec}})$ scaled by $1/\text{MU}$. A natural candidate is the G_2 -invariant inner product of $\log(X_{\text{sec}}/X_1)$ with X_{sec}^{-1} under the $J_3(\mathbb{O})$ trace form, but no identification is currently established.

7 Summary

The main results are:

1. **The Jordan triple product intertwiner $\Phi = \{e_3, \cdot, e_3\}$ vanishes on V_{ω^2}** (Proposition 2.1). This is forced by T -equivariance (Remark after the proposition), not just Peirce arithmetic: the map was never a $V_{\omega^2} \rightarrow V_1$ intertwiner. OP-B' of Addendum 45 is closed as algebraically impossible.
2. **Conjecture B $\Leftrightarrow m_t/m_c = \text{Tr}_{J_3(\mathbb{O})}(X_{\text{sec}})$** (Proposition 3.3), where $X_{\text{sec}} = \text{diag}(\pi, \pi^2, 4\pi^3)$ and $\text{Tr}(X_{\text{sec}}) = \mu_0 = \alpha^{-1}$ exactly (Proposition 3.2).
3. **Conjecture D $\Leftrightarrow E_d = \ln(\text{Tr}_{J_3(\mathbb{O})}(X_1))/\text{MU}$** (Proposition 4.3), where $X_1 = \text{diag}(2\pi/3, 3\pi^2/4, 16\pi^3/5)$ and $\text{Tr}(X_1) = \mu_1$ exactly (Proposition 4.2).
4. **The factorization $X_1 = X_{\text{sec}} \circ K$** (Proposition 4.5) identifies the Peirce center matrix $K = \text{diag}(2/3, 3/4, 4/5)$ as the natural $J_3(\mathbb{O})$ object relating the two conjectures. Its eigenvalues $n/(n+1)$ are the sector-power centers, fixed by the G_2 -invariant Peirce decomposition.
5. **The G_0 – G_1 link $G_0 - G_1 = -\ln(\text{MU})/\text{MU}$ is exact** (Theorem 5.2). It reads in $J_3(\mathbb{O})$ language as $G_0 - G_1 = \ln(\text{Tr}(X_{\text{sec}})/\text{Tr}(X_1))/\text{MU}$. Corollary 5.3: OP-D reduces to OP-B plus identifying $-\ln(\text{MU})/\text{MU}$ as a $J_3(\mathbb{O})$ intrinsic invariant.
6. **Route 3 (spectral measure) is the most natural path to OP-B**: treat ρ as the $J_3(\mathbb{O})$ spectral density of X_{sec} ; then $\text{Tr}(X_{\text{sec}})$ is its normalization, and Conjecture A (already structural) supplies the charm anchor.

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Table 2: Updated status of Conjectures B and D after this addendum.

	Conjecture B	Conjecture D
Statement	$m_t/m_c = \mu_0 = \alpha^{-1}$	$E_d = \ln(\mu_1)/\text{MU}$
$J_3(\mathbb{O})$ form	$m_t/m_c = \text{Tr}(X_{\text{sec}})$	$E_d = \ln(\text{Tr}(X_1))/\text{MU}$
Key element	$X_{\text{sec}} = \text{diag}(\pi, \pi^2, 4\pi^3) \in V_1$	$X_1 = \text{diag}(\frac{2\pi}{3}, \frac{3\pi^2}{4}, \frac{16\pi^3}{5}) \in V_1$
Trace identity	$\text{Tr}(X_{\text{sec}}) = \mu_0$ (exact)	$\text{Tr}(X_1) = \mu_1$ (exact)
Mass/energy error	+1.1% ratio; +2.0% m_t	−2.2% m_d
Previous target	$\ \{e_3, \cdot, e_3\}\ ^2 = \mu_0$ (OP-B')	G_0 – G_1 link
New status	OP-B' closed (intertwiner $\equiv 0$); replaced by trace target	Reduces to OP-B + $-\ln(\text{MU})/\text{MU}$
Remaining input	$\mathbb{Z}_3$ -spectral normalization of m_t/m_c	$-\ln(\text{MU})/\text{MU}$ as $J_3(\mathbb{O})$ Casimir

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