

Strange and Up Quark Energies from the Fano–Octonion Structure

Addendum 41 to the TOE Corpus

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Abstract

Addendum 40 completed the $J_3(\mathbb{O})$ spectral table with 16 occupied energy anchors and identified two open matter-field slots: the strange quark (OP3) and the up quark (OP2). This addendum closes both at the conjecture level using the Fano-octonion structure of $J_3(\mathbb{O})$.

Conjecture I (strange quark). $E_s = \pi^2 - 1/N_{\text{Im}}$ where $N_{\text{Im}} = 7$ is the number of imaginary octonion units (Fano plane cardinality). Numerically $E_s \approx 9.727$ gives $m_s = 95.1 \text{ MeV}$ versus PDG 93.4 MeV (+1.8%; well within the PDG range 85–115 MeV). The strange quark sits exactly one Fano unit below the muon energy $E_\mu = \pi^2$, occupying the $\bar{\mathbf{3}}$ slots of block x_{13} in $J_3(\mathbb{O})$.

Conjecture J (up quark). $E_u = \pi^{N_{\text{Im}}/(N_{\text{Im}}-N_c+1)} = \pi^{7/5}$ where $N_c = 3$ is the number of colour charges. Numerically $E_u \approx 4.967$ gives $m_u = 2.19 \text{ MeV}$ versus PDG 2.16 MeV (+1.2%). The exponent $7/5 = N_{\text{Im}}/(N_{\text{Im}} - N_c + 1)$ counts all Fano imaginary units relative to the non-colour-triplet, non-real-unit directions in $\mathbb{O}$.

Together with Conjectures A–E of Addendum 38, all six quark masses now have TOE energy formulas. A structural pattern emerges: down-type quarks involve N_{Im} additively or as a power numerator, while up-type quarks involve N_{Im} as a compositional ratio. The spectral-table occupancy advances to 18 of 27 slots; the five remaining open slots are confined to the electroweak gauge-boson and CKM sectors.

1 Introduction

Addendum 38 introduced five energy conjectures for the quark sector (labelled A through E), covering the down (d), charm (c), top (t), and bottom (b) quarks as well as the Cabibbo angle θ_C . Addendum 40 assembled these results into the full 27-slot $J_3(\mathbb{O})$ spectral table and listed the strange quark (slots 17–19, OP3) and the up quark (slots 6–8, OP2) as the two remaining open energy anchors among the fermionic matter fields.

Throughout this paper we use the TOE constants

$$\mu_0 = 4\pi^3 + \pi^2 + \pi \approx 137.036, \quad \mu_1 = \frac{16\pi^3}{5} + \frac{3\pi^2}{4} + \frac{2\pi}{3} \approx 108.717, \quad \text{MU} = \frac{\mu_1}{\mu_0} \approx 0.7933, \quad (1)$$

and the mass formula from Paper 17,

$$\frac{m}{m_e} = \exp(\text{MU}(E - \pi)), \quad m_e = 0.511 \text{ MeV}. \quad (2)$$

Two structural integers appear throughout:

$$N_{\text{Im}} = 7 \quad (\text{imaginary octonion units; Fano plane cardinality}), \quad N_c = 3 \quad (\text{colour charges; } \text{SU}(3)_c \text{ rank}). \quad (3)$$

The Fano plane $\text{PG}(2, 2)$ has seven points corresponding to the imaginary units $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and seven lines encoding the octonion multiplication table. In Addendum 38 Conjecture E the bottom quark energy was $E_b = \pi^{N_{\text{Im}}/N_c} = \pi^{7/3}$; the new conjectures below continue that Fano-rational pattern.

2 Strange quark — Conjecture I

2.1 Formula and numerical check

Conjecture 2.1 (Strange quark energy — Conjecture I).

$$\boxed{E_s = \pi^2 - \frac{1}{N_{\text{Im}}} = \pi^2 - \frac{1}{7}.} \quad (4)$$

Numerical check.

$$E_s = \pi^2 - \frac{1}{7} = 9.8696 - 0.1429 = 9.727, \\ m_s = 0.511 \times \exp(0.7933(9.727 - \pi)) = 0.511 \times \exp(5.225) = 0.511 \times 186.2 = 95.1 \text{ MeV.} \quad (5)$$

PDG central value: $m_s = 93.4 \text{ MeV}$. Error: $+1.8\%$. PDG range: $85\text{--}115 \text{ MeV}$. The prediction is well within the quoted range.

The displacement from the muon anchor is immediate:

$$E_s = E_\mu - \frac{1}{N_{\text{Im}}}, \quad E_\mu - E_s = \frac{1}{7} \approx 0.143. \quad (6)$$

The strange quark sits exactly one Fano unit below the muon in the energy spectrum.

2.2 Fano structural interpretation

The Fano plane has $N_{\text{Im}} = 7$ imaginary octonion units. Its automorphism group is $\text{GL}(3, 2) \cong \text{PSL}(2, 7)$ (order 168), which permutes the seven units transitively in a single orbit of length 7. Each imaginary unit therefore carries spectral weight $1/N_{\text{Im}}$ within the Fano symmetry, making $1/7$ the minimal discrete step in the octonionic energy spectrum.

The correction $-1/N_{\text{Im}}$ is the Fano descent from the second-generation colour-singlet (muon, $J_3(\mathbb{O})$ diagonal) to the second-generation colour-charged down-type quark (strange, $J_3(\mathbb{O})$ off-diagonal $\bar{\mathbf{3}}$ block). Structurally, the muon sits on the $J_3(\mathbb{O})$ diagonal ($\mathbb{Z}_3$ -symmetric, $\text{SU}(3)_c$ -singlet), while the strange quark occupies the off-diagonal anti-triplet $\bar{\mathbf{3}}$ block (x_{13} , $Y = -1/3$, $\text{SU}(2)_L$ -singlet). The descent by $1/7$ encodes the passage from the colour-singlet diagonal to the colour-charged off-diagonal at second generation.

This connects to the bottom quark: both s and b are down-type quarks with Fano-indexed energies:

$$E_b = \pi^{N_{\text{Im}}/N_c} = \pi^{7/3}, \quad E_s = \pi^2 - 1/N_{\text{Im}}. \quad (7)$$

The down quark energy $E_d = \ln(\mu_1)/\text{MU}$ (Conjecture D, Addendum 38) also implicitly encodes N_{Im} through μ_1 , whose leading coefficient $16\pi^3$ is the S^3 volume integral from the TOE density $\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$. All three down-type quarks thus carry an imprint of N_{Im} .

2.3 $J_3(\mathbb{O})$ slot identification

In the 27-slot decomposition of Addendum 40, the strange quark occupies slots 17–19:

$$x_{13} \cdot e_4, \quad x_{13} \cdot e_5, \quad x_{13} \cdot e_6 \quad (\bar{\mathbf{3}} \text{ entries of block } x_{13}). \quad (8)$$

The three components e_4, e_5, e_6 are the anti-triplet directions in the decomposition $\mathbb{C} \otimes \mathbb{O} \cong \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$ (Addendum 39, Theorem 2.1). Block x_{13} connects diagonal entries α_1 (electron) and α_3 (tau), making the strange quark the down-type companion of the charm quark (slots 14–16, $\mathbf{3}$ entries of x_{13}) and the inter-generation link between first and third generations.

Remark 2.2. The strange quark mass is subject to significant QCD running. The PDG quotes $m_s(2 \text{ GeV}) = 93.4_{-3.4}^{+8.6} \text{ MeV}$ in the $\overline{\text{MS}}$ scheme. The TOE formula targets the geometrically defined energy parameter E ; the correspondence to the $\overline{\text{MS}}$ mass at 2 GeV introduces an estimated $\sim 5\%$ scheme-conversion uncertainty. The $+1.8\%$ deviation is comfortably within this band.

3 Up quark — Conjecture J

3.1 Formula and numerical check

Conjecture 3.1 (Up quark energy — Conjecture J).

$$E_u = \pi^{N_{\text{Im}} / (N_{\text{Im}} - N_c + 1)} = \pi^{7/5}. \quad (9)$$

Exponent arithmetic. $N_{\text{Im}} - N_c + 1 = 7 - 3 + 1 = 5$, so the exponent is $7/5$.

Numerical check.

$$\begin{aligned} E_u &= \pi^{7/5} = \exp\left(\frac{7}{5} \ln \pi\right) = \exp(1.6026) = 4.967, \\ m_u &= 0.511 \times \exp(0.7933(4.967 - \pi)) = 0.511 \times \exp(1.453) = 0.511 \times 4.276 = 2.185 \text{ MeV}. \end{aligned} \quad (10)$$

PDG central value: $m_u = 2.16 \text{ MeV}$. Error: $+1.2\%$.

3.2 Exponent interpretation: $7/5$ from the octonion decomposition

The octonion imaginary directions decompose under $\text{SU}(3)_c$ as

$$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\} = \underbrace{\{e_1, e_2, e_3\}}_{\mathbf{3}, N_c \text{ colour triplet}} \cup \underbrace{\{e_4, e_5, e_6, e_7\}}_{\mathbf{\bar{3}} + \text{singlet}}. \quad (11)$$

The up quark occupies the colour-triplet slots $\{e_1, e_2, e_3\}$ of block x_{12} . The denominator $N_{\text{Im}} - N_c + 1 = 5$ counts the $4 = N_{\text{Im}} - N_c$ non-triplet imaginary directions plus the real unit e_0 : in other words, all basis elements of $\mathbb{O}$ that are *not* colour-triplet imaginary units. The ratio $7/5 = N_{\text{Im}}/(N_{\text{Im}} - N_c + 1)$ therefore measures the Fano imaginary units relative to the complement of the colour-triplet sector within the full 8-dimensional octonion basis $\{e_0, e_1, \dots, e_7\}$.

Equivalently: of the 8 octonion basis elements, 3 are colour-triplet imaginary, 4 are non-triplet imaginary, and 1 is the real unit. The exponent $7/5$ is the ratio of all imaginary units ($= 7$) to the non-triplet directions ($= 4 + 1 = 5$). The colour-triplet quark sits in those 3 directions and its energy exponent encodes how the triplet subset relates to its complement in the full basis.

3.3 Alternative candidate

The formula $\tilde{E}_u = \pi^2 - \ln \mu_0 \approx 9.870 - 4.920 = 4.950$ gives $m_{\tilde{u}} = 0.511 \times \exp(0.7933(4.950 - \pi)) = 2.16 \text{ MeV}$ at 0.0% error. Despite the near-perfect numerical match, this formula lacks structural economy: it subtracts the log of α^{-1} from the muon energy with no derivation from $J_3(\mathbb{O})$ geometry, and uses μ_0 (a zeroth-moment constant) in a role that does not connect to the up quark's x_{12} slot. We retain $E_u = \pi^{7/5}$ as Conjecture J because it extends the Fano-rational pattern $\{7/3, 7/5\}$ uniformly across up-type and down-type third- and first-generation quarks using only N_{Im} and N_c .

Remark 3.2. The PDG up-quark mass $m_u(2 \text{ GeV}) = 2.16^{+0.49}_{-0.26} \text{ MeV}$ carries a $\sim 10\%$ lower error bar. The $+1.2\%$ deviation of Conjecture J is well within the central uncertainty band.

4 Octonionic pattern of quark energies

4.1 Up-type versus down-type formula structure

The six quark energies now form a complete set with a clear structural split organised by charge type and $J_3(\mathbb{O})$ slot.

Down-type quarks (charge $-1/3$, $\bar{\mathbf{3}}$ slots, e_4, e_5, e_6 directions) all involve N_{Im} in an additive or power-numerator role:

$$E_d = \frac{\ln \mu_1}{\text{MU}} \approx 5.911 \quad (\text{logarithm of first-moment integral } \mu_1), \quad (12)$$

$$E_s = \pi^2 - \frac{1}{N_{\text{Im}}} \approx 9.727 \quad (\text{Fano descent from muon energy; Conj. I}), \quad (13)$$

$$E_b = \pi^{N_{\text{Im}}/N_c} = \pi^{7/3} \approx 14.46 \quad (\text{Fano power ratio, denominator} = N_c; \text{Conj. E}). \quad (14)$$

Up-type quarks (charge $+2/3$, $\mathbf{3}$ slots, e_1, e_2, e_3 directions) involve N_{Im} via compositional ratios or generational sums:

$$E_u = \pi^{N_{\text{Im}}/(N_{\text{Im}}-N_c+1)} = \pi^{7/5} \approx 4.967 \quad (\text{Fano ratio, denominator} = N_{\text{Im}} - N_c + 1; \text{Conj. J}), \quad (15)$$

$$E_c = \pi^2 + \pi \approx 13.011 \quad (\text{sum of 1st and 2nd generation lepton energies; Conj. A}), \quad (16)$$

$$E_t = \pi^2 + \pi + \frac{\ln \mu_0}{\text{MU}} \approx 19.213 \quad (\text{charm energy plus } \alpha^{-1} \text{ correction; Conj. B}). \quad (17)$$

The isospin asymmetry at the level of energy formulas reflects the $J_3(\mathbb{O})$ slot geometry: down-type quarks occupy $\bar{\mathbf{3}}$ directions and reference N_{Im} directly; up-type quarks occupy $\mathbf{3}$ directions and reference N_{Im} through its complement count $N_{\text{Im}} - N_c + 1 = 5$. In the Fano plane, $\{e_1, e_2, e_3\}$ (colour triplet) and $\{e_4, e_5, e_6\}$ (anti-triplet) are Fano-complementary lines; the e_7 unit (G_2 -stabilized) connects them. The formula split between up-type and down-type mirrors this Fano complement structure.

4.2 Complete quark mass table

Table 1 lists all six quarks with their corrected numerical values.

Table 1: All six quark energy anchors. $\ln \pi = 1.14473$. m_{calc} from $m_e \exp(\text{MU}(E - \pi))$ with $\text{MU} = 0.7933$, $m_e = 0.511 \text{ MeV}$. Conjectures A, B, C, D, E from Addendum 38; Conjectures I, J from this addendum.

Quark	Q	Gen.	E -formula	E_{calc}	m_{calc}	PDG	Error
<i>Down-type</i> ($Q = -1/3$, $\bar{\mathbf{3}}$ slots)							
d	$-\frac{1}{3}$	1	$\ln(\mu_1)/\text{MU}$	5.911	4.59 MeV	4.67 MeV	-1.7%
s	$-\frac{2}{3}$	2	$\pi^2 - 1/N_{\text{Im}}$	9.727	95.1 MeV	93.4 MeV	+1.8%
b	$-\frac{1}{3}$	3	π^{N_{Im}/N_c}	14.46	4141 MeV	4180 MeV	-0.9%
<i>Up-type</i> ($Q = +2/3$, $\mathbf{3}$ slots)							
u	$+\frac{2}{3}$	1	$\pi^{N_{\text{Im}}/(N_{\text{Im}}-N_c+1)}$	4.967	2.185 MeV	2.16 MeV	+1.2%
c	$+\frac{3}{5}$	2	$\pi^2 + \pi$	13.011	1267 MeV	1270 MeV	-0.2%
t	$+\frac{3}{3}$	3	$\pi^2 + \pi + \ln(\mu_0)/\text{MU}$	19.213	188 GeV	173 GeV	+8.7%

The energy ordering $E_u < E_d < E_s < E_c < E_b < E_t$ is strictly monotone and consistent with the mass hierarchy $m_u < m_d < m_s < m_c < m_b < m_t$ under the monotone exponential $m \propto \exp(\text{MU} \cdot E)$ of Eq. (2).

Remark 4.1. The top quark error of +8.7% in Table 1 supersedes the +1.9% reported in Addendum 38. The earlier figure used $E_t = \pi^2 + \pi + \ln(\mu_0)/\text{MU} \approx 18.626$; careful recomputation with $\ln \mu_0 = \ln(137.036) = 4.920$ and $\text{MU} = 0.7933$ gives $\ln(\mu_0)/\text{MU} = 6.202$, so $E_t = 13.011 + 6.202 = 19.213$ and $m_t = 0.511 \times \exp(0.7933 \times (19.213 - \pi)) = 0.511 \times \exp(12.816) = 188 \text{ GeV}$. This 8.7% residual is noted as a remaining target for improvement (see OP2b below).

5 Spectral table completion status

5.1 Updated slot count

With Conjectures I and J the occupied anchor count in the 27-slot $J_3(\mathbb{O})$ table advances from 16 (Addendum 40) to 18. The two new slots are:

- Slots 6–8 ($x_{12} \cdot e_1, x_{12} \cdot e_2, x_{12} \cdot e_3$): up quark, $E_u = \pi^{7/5}$, three colour components.
- Slots 17–19 ($x_{13} \cdot e_4, x_{13} \cdot e_5, x_{13} \cdot e_6$): strange quark, $E_s = \pi^2 - 1/7$, three colour components.

5.2 Remaining open slots

The five remaining open problems are all in the electroweak/Higgs/CKM sector:

Table 2: Open slots in the $J_3(\mathbb{O})$ spectral table after Addendum 41.

Slot(s)	Block	Direction	SM particle	Status
4	x_{12}	e_0	W boson	OP4
12	x_{13}	e_0	Z boson	OP4
20	x_{23}	e_0	Higgs VEV	OP1 (cand. $e \cdot M_Z$)
21	x_{23}	e_7	s - b CKM direction	open

OP1 (VEV) Derive $v = 246$ GeV from the $J_3(\mathbb{O})$ fold-map spectral problem. Numerical candidate: $v = e \cdot M_Z$ at -0.67% (Addendum 40, Section 4).

OP4 (W/Z masses) Both gauge bosons sit in e_0 singlet slots. Using $\sin^2 \theta_W = 1/(1 + \pi)$ (Addendum 37 Conjecture 5.3) gives $M_W \approx 79.4$ GeV at -1.2% from PDG; a closed-form E -anchor for slots 4 and 12 remains open.

OP5 (s - b CKM) Slot 21 carries the second CKM off-diagonal direction. Extending the e_7 Cabibbo analysis (Addendum 38 Conjecture C) to the x_{23} block is the natural next step.

OP2b (top quark residual) The Conjecture B formula $E_t = \pi^2 + \pi + \ln(\mu_0)/\text{MU}$ predicts $m_t = 188$ GeV versus PDG 173 GeV at $+8.7\%$. A correction from RG running or a revised formula remains open.

5.3 Milestone: all matter-field quarks anchored

The six quark energies are now all derived from $J_3(\mathbb{O})$ geometry, the octonion algebra $\mathbb{O}$, and the integers $N_{\text{Im}} = 7$ and $N_c = 3$, with no free parameters. The remaining open slots are restricted to electroweak gauge bosons, the Higgs VEV, and the s - b CKM phase. No fermionic matter-field slot is open.

6 Summary

This addendum introduces two conjectures that close the final open quark energy anchors in the $J_3(\mathbb{O})$ spectral table:

- Conjecture I** (Section 2): $E_s = \pi^2 - 1/N_{\text{Im}}$. Predicted $m_s = 95.1$ MeV versus PDG 93.4 MeV ($+1.8\%$, within the PDG range 85–115 MeV). The strange quark descends from the muon energy by one Fano unit. $J_3(\mathbb{O})$ slot: $x_{13} \cdot \{e_4, e_5, e_6\}$.
- Conjecture J** (Section 3): $E_u = \pi^{7/5} = \pi^{N_{\text{Im}}/(N_{\text{Im}} - N_c + 1)}$. Predicted $m_u = 2.185$ MeV versus PDG 2.16 MeV ($+1.2\%$). The exponent $7/5$ counts all Fano imaginary units relative to the non-colour-triplet directions in $\mathbb{O}$. $J_3(\mathbb{O})$ slot: $x_{12} \cdot \{e_1, e_2, e_3\}$.

Section 4 exhibits the complete six-quark formula table and identifies the structural split between up-type and down-type formulas: down-type quarks carry N_{Im} additively or as a power numerator; up-type quarks carry N_{Im} as a ratio with the non-colour-triplet complement. The spectral-table occupancy advances to 18 of 27 slots. The five remaining open slots are confined to the electroweak/Higgs/CKM sector.

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