

Addendum 396: The Quark CP Phase at NLO

The δ_{CKM} angle is within experimental error; the Jarlskog NLO closure reduces to a G_2 identity, advanced through the associative 3-form (Off-diagonal NLO follow-up to Addendum 395; Addenda 66, 81, 82, 91, 92)

Léon Fernando Vlegels

2026-06-22

Abstract

Addendum 395 found the quark off-diagonal CP phase $\delta_{\text{CKM}} = \pi/3 + \theta_C/2 = 66.43^\circ$ a 1.3% hit. Does the residual close at NLO? **The angle is not the problem.** 66.43° lies within PDG $\gamma = 65.9 \pm 3.3^\circ$ at 0.16σ — the “1.3%” against the central value is within experimental error, not a tension — and Addendum 92 shows the angle is pinned at the sine ceiling ($\sin \delta_{\text{CKM}} = 0.916$, $0.916 \times 1.118 > 1$), so it cannot move to absorb any correction. **The genuine NLO item is the Jarlskog** $J_{\text{CKM}} = A^2 \lambda^6 \bar{\eta}$. Addendum 91 propagated the NLO Wolfenstein $A_{\text{NLO}} = A_{\text{LO}}(1 - 4\lambda^2/5)$ and *worsened* J from -0.5% to -8.2% . Addendum 92’s paired-correction theorem restores it: if the (2,3) Peirce block carries $(1 - 4\lambda^2/5)^2$ and the (1,3) block the compensation $(1 + 8\lambda^2/5)$, then $Q = A^2 \Phi$ is invariant to $O(\lambda^2)$ — numerically the product is $1 - 0.48\%$, an $O(\lambda^4)$ residual — restoring J_{CKM} at NLO. **This is conditional on a G_2 identity** (conj:g2-pairing, OP-P92-1): that $A^2 \Phi / \sin \delta_{\text{CKM}}$ is invariant under the Peirce-block Jordan loop, with the required (1,3) orbit dimension $5/8$ non-integer, i.e. a structural identity, not a single loop. **We advance it.** The Jarlskog — the unitarity-triangle area — is built from the G_2 -invariant associative calibration 3-form φ (Addendum 66: the dihedral is $\arcsin(s_1 s_2 \varphi(\hat{v}_1, \hat{v}_2, e_7))$). Since $G_2 = \text{Stab}(\varphi)$, any quantity built from φ on the off-diagonal triple is G_2 -invariant under off-diagonal rotation — the structural reason the conjecture should hold. **The honest gap:** that the *specific* paired rescalings are *forced* by the φ -preserving structure (not merely algebraically cancel) is unproven; OP-P92-1 stays open, now sharpened. So the angle is consistent and the Jarlskog NLO closure is reduced and motivated, not completed. All claims pinned by `verify_P396.py` (7/7).

1 The angle is within experimental error

Proposition 1.1 (No angle tension; the angle is pinned). $\delta_{\text{CKM}} = 31\pi/84 = 66.43^\circ$ lies 0.16σ from the PDG unitarity-triangle angle $\gamma = 65.9 \pm 3.3^\circ$. The “1.3%” discrepancy quoted against the central 65.6° is well within the experimental uncertainty: there is no angle tension to close. Moreover (Addendum 92) the angle is pinned at the sine ceiling, $\sin \delta_{\text{CKM}} = 0.916$ with $0.916 \times 1.118 > 1$, so a shift in δ_{CKM} cannot supply the NLO correction to the Jarlskog. The angle is both consistent and fixed.

2 The Jarlskog: paired correction and the G_2 identity

Proposition 2.1 (The NLO worsening and the paired restoration). Addendum 91 propagated $A_{\text{NLO}} = A_{\text{LO}}(1 - 4\lambda^2/5)$ into $J = A^2 \lambda^6 \bar{\eta}$, worsening the residual to -8.2% . Addendum 92’s paired-correction theorem restores it: with the (2,3) block carrying $(1 - 4\lambda^2/5)^2$ and the (1,3) block the

compensation $(1 + 8\lambda^2/5)$,

$$\left(1 - \frac{4\lambda^2}{5}\right)^2 \left(1 + \frac{8\lambda^2}{5}\right) = 1 + O(\lambda^4) = 1 - 0.48\%, \quad (1)$$

so $Q = A^2\Phi$ is invariant to $O(\lambda^2)$ and J_{CKM} is restored at NLO to within an $O(\lambda^4)$ residual.

Theorem 2.2 (The closure reduces to a G_2 identity, and the identity is structurally motivated). *The restoration of Proposition 2.1 is conditional on **conj:g2-pairing** (OP-P92-1): that $A^2\Phi/\sin\delta_{\text{CKM}}$ is invariant under the Peirce-block Jordan loop as a G_2 structural identity (the required $(1, 3)$ orbit dimension is $5/8$, non-integer — not a single Jordan loop). This addendum advances it. The Jarlskog invariant is the area of the unitarity triangle, and Addendum 66 builds the CP structure from the G_2 -invariant associative calibration 3-form φ : the dihedral is $\phi = \arcsin(s_1 s_2 \varphi(\hat{v}_1, \hat{v}_2, e_7))$. Because $G_2 = \text{Stab}(\varphi)$, any quantity built from φ on the off-diagonal octonion triple is invariant under G_2 rotation of that triple. Hence the unitarity-triangle area, hence J_{CKM} , is G_2 -invariant under off-diagonal rotation — the structural reason **conj:g2-pairing** should hold.*

Remark 2.3 (The honest gap). Theorem 2.2 motivates the conjecture; it does not prove it. What remains is to show that the *specific* paired NLO rescalings $(1 - 4\lambda^2/5)^2$ and $(1 + 8\lambda^2/5)$ realize (are forced by) a φ -preserving G_2 transformation of the off-diagonal triple, rather than merely cancelling algebraically. The φ -invariance argument shows that *some* G_2 rotation preserves J ; it does not yet show these block rescalings are that rotation. OP-P92-1 therefore stays open, sharpened to this question.

Status

The quark off-diagonal CP phase at NLO, examined honestly: the *angle* $\delta_{\text{CKM}} = 66.43^\circ$ is within PDG $\gamma = 65.9 \pm 3.3^\circ$ (0.16σ , Proposition 1.1) — the “1.3%” is within experimental error, not a tension — and is pinned at the sine ceiling, so it neither needs nor can absorb a shift. The genuine NLO item is the Jarlskog J_{CKM} (Addendum 91’s -8.2%), restored by Addendum 92’s paired correction to an $O(\lambda^4) \sim 0.5\%$ residual (Proposition 2.1), conditional on **conj:g2-pairing** (OP-P92-1). This addendum advances the conjecture: J / the unitarity-triangle area is built from the G_2 -invariant associative 3-form φ , and $G_2 = \text{Stab}(\varphi)$ preserves φ -built quantities, so J is G_2 -invariant under off-diagonal rotation (Theorem 2.2). The honest gap is that the specific paired rescalings are shown consistent with, not yet forced by, the φ -structure (Remark 2.3). An honest partial: the angle is consistent, the Jarlskog NLO closure is reduced to a structurally-motivated G_2 identity but not completed. No published number changes; OP-P92-1 remains open, sharpened. Verifier: `verify_P396.py`, 7/7 pass.