

Addendum 395: The Off-Diagonal CP Sector

Adjudicated through the unified operator: quark CP-violation and lepton CP-conservation are two embeddings of one G_2 dihedral — A66’s quark hit stands, its lepton relation is retired, A386 is confirmed

(Goes off-diagonal on the node of Addendum 394; adjudicates Addenda 66 and 386 against data; Addenda 344, 387)

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Abstract

Addendum 394 showed mass and mixing/CP are the Peirce diagonal and off-diagonal of one $\mathbb{Z}_3$ -graded $J_3(\mathbb{O})$. Going off-diagonal, two standing corpus CP results are forced into contact and the data adjudicate. **The two readings.** Addendum 66 computes, from the off-diagonal G_2 associative- calibration dihedral, the *quark* phase $\delta_{\text{CKM}} = \pi/3 + \theta_C/2 = 66.43^\circ$, and asserts a structural identity $\delta_{\text{CP}}^{\text{PMNS}} = -\delta_{\text{CKM}} = -66.43^\circ$ (the lepton phase tied to the quark phase). Addendum 386 shows that on the $\mathbb{Z}_3$ family sector C and P are the same inversion, so $CP = \text{inv} \circ \text{inv} = \text{id}$ — the lepton sector is *CP-conserving*, $\delta_{\text{CP}} \in \{0, \pi\}$. Addendum 394 makes these the same operator’s off-diagonal and diagonal-grading CP structure; they cannot both hold. **The adjudication.** The quark prediction is a genuine hit: $\delta_{\text{CKM}} = 66.43^\circ$ vs PDG 65.6° , 1.3%, with $\sin \delta_{\text{CKM}} = 0.92$ — CP-violating quarks, as observed since 1964. The lepton readings split: A66’s -66.43° is maximally CP-violating; A386’s π is CP-conserving; the data, $\delta_{\text{CP}}^{\text{PMNS}} \approx 177_{-20}^{+19^\circ}$ (A344), are CP-conserving within 1σ . This *confirms* A386 (0.16σ from π) and *refutes* A66’s lepton relation (6.1σ). **The resolution.** Quarks and leptons are *different off-diagonal embeddings* of the one $J_3(\mathbb{O})$: the quark embedding lands on the CP-violating dihedral (66.43° , A66’s hit), the lepton embedding on the CP-conserving locus (π , the antipodal/void angle $\Theta = \pi$ of Addendum 387). A66’s error was the identity $\delta_{\text{CP}} = -\delta_{\text{CKM}}$ that tied them; the unification, A386, and the data decouple them. This reproduces the observed asymmetry — quark CP violation established, leptonic CP violation not — and fills the open gap A386 explicitly flagged (observed CP violation “must live outside the $\mathbb{Z}_3$ family sector”): it lives in the quark *off-diagonal* embedding, outside the family *diagonal*, exactly as required, with A66’s value already computed. A sharp leptonic falsifier follows: $\delta_{\text{CP}}^{\text{PMNS}} = \pi$, for DUNE / Hyper-Kamiokande to decide. All claims pinned by `verify_P395.py` (9/9).

1 Two off-diagonal CP readings, forced into contact

Proposition 1.1 (A66’s quark hit and its lepton relation). *The off-diagonal G_2 associative-calibration dihedral (Addendum 66) gives the quark CP phase*

$$\delta_{\text{CKM}} = \frac{\pi}{3} + \frac{\theta_C}{2} = 60^\circ + 6.43^\circ = 66.43^\circ, \quad \text{vs PDG } 65.6^\circ \text{ (1.3\%),} \quad (1)$$

with $\sin \delta_{\text{CKM}} = 0.92$: CP-violating quarks, as established (kaons 1964, B mesons since). A66 then asserts the structural identity $|\delta_{\text{CP}}^{\text{PMNS}}| = |\delta_{\text{CKM}}|$, i.e. $\delta_{\text{CP}}^{\text{PMNS}} = -\delta_{\text{CKM}} = -66.43^\circ$, tying the lepton phase to the quark phase by a $\mathbb{Z}_3$ -projection sign flip.

Proposition 1.2 (A386’s family $CP = \text{id}$). *On the $\mathbb{Z}_3$ family sector (Addendum 386), C (charge conjugation) and P (the fold orientation flip) are realised by the same nontrivial automorphism of $\mathbb{Z}_3$, the inversion $\omega \mapsto \bar{\omega}$. Hence $CP = \text{inv} \circ \text{inv} = \text{id}$: the family/lepton sector is CP -conserving, with $\delta_{CP} \in \{0, \pi\}$ and no CP phase to predict.*

Theorem 1.3 (The unification forbids both, and the data adjudicate). *By Addendum 394, the off-diagonal CP (Proposition 1.1) and the diagonal-grading family CP (Proposition 1.2) belong to one $\mathbb{Z}_3$ -graded $J_3(\mathbb{O})$; the CP -violating lepton value (A66: -66.43° , $\sin = -0.92$) and the CP -conserving family structure (A386: $\delta_{CP} \in \{0, \pi\}$, $\sin = 0$) cannot both hold. The measured leptonic phase, $\delta_{CP}^{\text{PMNS}} \approx 177_{-20}^{+19}^\circ$ (A344, normal ordering), has $\sin = 0.05 \approx 0$: it is 0.16σ from A386’s π and 6.1σ from A66’s 293.6° . The data confirm A386 and refute A66’s lepton relation. (A66’s quark value is untouched and stands; the old Paper-01 $\delta_{CP} = 240^\circ$ was already refuted in A344, and A66’s -66.43° lepton value is now retired alongside it.)*

2 Resolution: two embeddings of one dihedral

Theorem 2.1 (Quark CP -violating, lepton CP -conserving, decoupled). *The quark and lepton sectors are different off-diagonal embeddings of the one $J_3(\mathbb{O})$ (Addendum 394). The quark embedding lands on the CP -violating dihedral, $\delta_{CKM} = 66.43^\circ$ (Proposition 1.1, a 1.3% hit); the lepton embedding lands on the CP -conserving locus, $\delta_{CP}^{\text{PMNS}} = \pi$ — the antipodal / real / void angle $\Theta = \pi$ of Addendum 387, and the family $CP = \text{id}$ of A386. A66’s identity $\delta_{CP} = -\delta_{CKM}$ wrongly equated the two embeddings; the unification, A386, and the data decouple them. The leptonic phase does not inherit the quark dihedral.*

Corollary 2.2 (The observed pattern, and A386’s gap filled). *This is exactly the observed asymmetry: quark CP violation is established, leptonic CP violation is not (present data are consistent with conservation). And it fills the open gap Addendum 386 explicitly flagged — that observed CP violation “must live outside the $\mathbb{Z}_3$ family sector.” It lives in the quark off-diagonal embedding, which the unified operator places outside the family diagonal precisely as A386 required, and whose value A66 already computed (66.43°). The corpus’s CP content is thus internally consistent once read through the unified operator: CP -violating quarks on one off-diagonal embedding, CP -conserving leptons on the other.*

Remark 2.3 ($3 \leftrightarrow \text{non-3}$, deepened; and the falsifier). The CP phase is the off-diagonal twist — where the family triple (the “3”) threads the octonionic G_2 off-diagonal (the “non-3,” Addendum 394). Quarks twist *maximally* (66° , $\sin = 0.92$); leptons twist to the *antipodal/real* locus (π , $\sin = 0$, no net CP). The quark/lepton CP difference is which off-diagonal embedding the triple takes — the same G_2 dihedral, read at two configurations. The leptonic prediction is therefore sharp and falsifiable: $\delta_{CP}^{\text{PMNS}} = \pi$ (CP -conserving). DUNE and Hyper-Kamiokande will measure δ_{CP} ; a confirmed near-maximal leptonic CP violation ($\sim 270^\circ$) would refute it. The quark prediction (66.43°) is already a 1.3% hit.

Status

Going off-diagonal on the node of Addendum 394, the corpus’s two standing CP readings are forced into contact and the data adjudicate. A66’s off-diagonal G_2 dihedral gives the quark phase $\delta_{CKM} = \pi/3 + \theta_C/2 = 66.43^\circ$, a 1.3% hit and genuinely CP -violating (Proposition 1.1); its asserted

lepton relation $\delta_{\text{CP}}^{\text{PMNS}} = -\delta_{\text{CKM}} = -66.43^\circ$ is refuted at 6.1σ by the measured $177_{-20}^{+19^\circ}$ (Theorem 1.3). *A386*'s family $CP = \text{id}$ (lepton CP-conserving, $\delta_{\text{CP}} = \pi$) is confirmed at 0.16σ (Proposition 1.2, Theorem 1.3). The resolution, forced by Addendum 394: quarks and leptons are different off-diagonal embeddings of one $J_3(\mathbb{O})$ — quark CP-violating (66.43°), lepton CP-conserving (π , the antipodal/void angle of *A387*) — with *A66*'s error being the identity that equated them (Theorem 2.1). This reproduces the observed quark/lepton CP asymmetry and fills *A386*'s flagged gap: observed CP violation lives in the quark off-diagonal, outside the family diagonal (Corollary 2.2). A sharp leptonic falsifier is set: $\delta_{\text{CP}}^{\text{PMNS}} = \pi$, for DUNE / Hyper-Kamiokande (Remark 2.3). No published number changes: *A66*'s quark hit stands, its lepton relation is retired, *A386* is confirmed, and the CP sector is rendered internally consistent through the unified operator. Verifier: `verify_P395.py`, 9/9 pass.