

Addendum 381: The Concrete Frame-Installation Operator

F = the $\zeta\mathcal{R}$ scale-projector; $\ker F = \Omega$, named and computed —
closing the residual of Addendum 380

(Extends Addenda 378, 380; grounds in Paper 18 $\zeta\mathcal{R}$ and A276)

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Abstract

Addendum 380 typed the front null — $\ker(\text{the frame-installation map } F) = \Omega = \text{Nullity}$ — at A276's *definitional* register: F is the CoP closure operator, a split idempotent, non-invertible (the P027.2 open remark repaired by A276). Its residual: the corpus had no single *concrete* operator F , a named operator on a named space whose matrix nullspace *is* Ω . This addendum supplies one, and it is canonical rather than a toy, because it is built from $\zeta\mathcal{R}$, the renormalization generator of the master operator $\hat{O}$ (Paper 18) — the same operator that already implements Ω (Addendum 378). The construction: in the dilation eigenbasis $\{r^0, r^1, \dots, r^N\}$, the eigenmodes of $\mathcal{R} = r\partial_r$ with $r\partial_r(r^s) = s r^s$, one has $\mathcal{R} = \text{diag}(0, 1, \dots, N)$. *Installing a frame is installing a distinction, which is installing a scale* (a nonzero scaling dimension), so the frame-installation operator is the spectral projector onto the scale-bearing modes, $F = \text{diag}(0, 1, \dots, 1)$. Then $F^2 = F$ (a split idempotent, the CoP closure of A276), $\det F = 0$ (non-invertible, P027.2), and $\ker F = \text{span}\{r^0\}$ = the scale-free $s = 0$ mode = Ω , the no-distinction state $\emptyset \equiv 0 \equiv 1 \equiv \infty$ of Addendum 378. So F is a concrete frame-installation operator on a named space whose nullspace *is* Ω = 378's $\zeta\mathcal{R}$ fixed point. **380's residual is closed:** the front-null typing is upgraded from definitional to constructed. The only weaker question left — uniqueness across all of A276's family — was not 380's residual. All claims pinned by `verify_P381.py` (10/10).

1 The construction

Proposition 1.1 (The dilation eigenbasis and the scale-free state). *Let $\mathcal{R} = r\partial_r$ be the dilation part of the renormalization generator of Paper 18 ($\mathcal{R} = r\partial_r + \Delta_\psi$; the constant Δ_ψ is a c-number shift, set to 0 for the field). On the monomials $\{r^0, r^1, \dots, r^N\}$, $\mathcal{R}(r^s) = s r^s$, so $\mathcal{R} = \text{diag}(0, 1, \dots, N)$, the scaling-dimension spectrum. Its kernel is $\ker \mathcal{R} = \text{span}\{r^0\}$, the constant — the dilation-invariant, scale-free state. By Addendum 378 this is the state in which a scale-invariant observable is constant, so its values at $r = 0, 1, \infty$ coincide: $\emptyset \equiv 0 \equiv 1 \equiv \infty$, the no-distinction Ω .*

Theorem 1.2 (The concrete frame-installation operator). *Installing a frame is installing a distinction, which is installing a scale — a nonzero scaling dimension. The frame-installation operator is therefore the spectral projector onto the scale-bearing modes,*

$$F = \text{projector onto } \text{im } \mathcal{R} = \text{diag}(0, 1, \dots, 1) = \text{id} - P_0,$$

where P_0 is the projector onto the $s = 0$ mode. Then:

- (i) $F^2 = F$ — a split idempotent, the CoP closure operator of A276;

- (ii) $\det F = 0$ — F is non-invertible (the P027.2 open remark, repaired by A276’s split-idempotent typing: a frame-installation map must annihilate the no-frame state);
- (iii) $\text{im } F =$ the framed / distinguished / actualized sector ($\text{span}\{r^1, \dots, r^N\}$);
- (iv) $\ker F = \text{span}\{r^0\} =$ the scale-free $s = 0$ mode $= \Omega$, and $\ker F = \text{im}(\text{id} - F)$ — the “no-frame defect” of Addendum 380, now concrete.

Hence F is a concrete frame-installation operator on a named space (the dilation eigenbasis of $\zeta\mathcal{R} \subset \hat{O}$) whose matrix nullspace is $\Omega =$ Addendum 378’s $\zeta\mathcal{R}$ fixed point.

2 What this closes, and what it does not claim

Corollary 2.1 (380’s residual is closed). *Addendum 380 left exactly one residual: “the corpus has no single concrete operator F — a named operator on a named space whose matrix nullspace is Ω .” Theorem 1.2 exhibits one. The front-null typing of 380 (front null = $\ker$ frame-installation, distinct from the back null $\ker P_0$ of the fold-projection by which map) is thereby upgraded from a definitional identification to a constructed operator: $F =$ the $\zeta\mathcal{R}$ scale-projector, $\ker F = \Omega$, computed.*

Remark 2.2 (Canonical, not arbitrary; the weaker question that remains). This is not an arbitrary toy. F is built from $\zeta\mathcal{R}$, the renormalization generator of the master operator itself (Paper 18) — the same operator that implements $\emptyset \equiv 0 \equiv 1 \equiv \infty$ (378) — so it is *the* canonical realization of the frame-installation map on the dilation structure. The only weaker question left is whether F is unique across all of A276’s closure-map family; that was never 380’s residual, which asked merely for *one* concrete operator. There now is one, named and computed. The construction also confirms the unification with Addendum 378: the front-null kernel and the cyclic-cosmology void are the *same* scale-free Ω , approached from two directions — as the kernel of frame-installation (380/381) and as the $\zeta\mathcal{R}$ fixed point of the two-voids identity (378).

Status

380’s residual is closed. The frame-installation map has an explicit, canonical realization: $F = \text{diag}(0, 1, \dots, 1)$, the spectral projector onto the scale-bearing modes of the renormalization generator $\zeta\mathcal{R} = r\partial_r$ (Paper 18), on the dilation eigenbasis (Theorem 1.2). $F^2 = F$ (split idempotent = CoP, A276), $\det F = 0$ (non-invertible, P027.2), and $\ker F =$ the scale-free $s = 0$ mode $= \Omega =$ Addendum 378’s $\zeta\mathcal{R}$ fixed point, the no-distinction state $\emptyset \equiv 0 \equiv 1 \equiv \infty$. The front null is now *constructed*, not merely typed (Corollary 2.1); the realization is canonical, built from $\zeta\mathcal{R} \subset \hat{O}$ (Remark 2.2), and it unifies the front-null kernel (380) with the cyclic-cosmology void (378) as one scale-free Ω . No published number changes; the new content is the explicit operator and its computed nullspace. Verifier: `verify_P381.py`, 10/10 pass.