

Addendum 378: The Two Voids Are One $\zeta\mathcal{R}$ Fixed Point

Solidifying heat-death = Ω by the master operator's
own renormalization generator

(Solidifies Addendum 377; grounds in Paper 18, Addenda 356a/365, Papers 04, A272)

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Abstract

Addendum 377 identified heat-death- $\emptyset$ (356a) with the pre-Bang Ω (Paper 27) as a synthesis — both are zero-distinction states — and left two seams: the two routes to the void (expansion $r \rightarrow \infty$ versus contraction $r \rightarrow 0$) were unwelded (its floor ii), and the identification was a structural reading rather than a theorem. This addendum discharges both with an operator already inside the master operator: the renormalization generator $\zeta\mathcal{R}$, with $\mathcal{R} = r\partial_r + \Delta_\psi$, which Paper 18 states (line 306) “implements the identity $\emptyset \equiv 0 \equiv 1 \equiv \infty$ under renormalization flow,” $e^{t\mathcal{R}}\psi(r) = e^{t\Delta_\psi}\psi(e^t r)$. The argument is short. The renormalization orbit of any $r_0 > 0$ under $r \mapsto e^t r$ is all of $(0, \infty)$, whose two limit points are $r \rightarrow 0$ and $r \rightarrow \infty$; so $0, 1, \infty$ lie on one orbit. A scale-invariant observable is constant ($r f'(r) = 0 \Rightarrow f' = 0$), so no scale-free observable distinguishes the orbit's points or its two ends — this *is* $\emptyset \equiv 0 \equiv 1 \equiv \infty$. The two voids are then the two ends of *one* orbit: heat-death- $\emptyset$ is the dilute end ($r \rightarrow \infty$), the fold's origin- $\emptyset$ (Paper 04) the dense end ($r \rightarrow 0$), and $\zeta\mathcal{R}$ identifies them. **This welds floor (ii)** — there was never a second void, only one orbit's two ends — and **upgrades heat-death = Ω from synthesis to a $\zeta\mathcal{R}$ fixed-point identity**, resting on a single named premise: that both ends are *scale-free* (heat death by 356a/365, the origin by Paper 04). The honest residual is complete scale-dissolution at heat death; the front-null typing inherits A272's candidate status. All claims pinned by `verify_P378.py` (10/10).

1 The operator and the identity

Proposition 1.1 ($\zeta\mathcal{R}$ and $\emptyset \equiv 0 \equiv 1 \equiv \infty$, made precise). *Let $\mathcal{R} = r\partial_r + \Delta_\psi$ be the renormalization generator of Paper 18, with flow $e^{t\mathcal{R}}\psi(r) = e^{t\Delta_\psi}\psi(e^t r)$. The dilation part has $r\partial_r(r^s) = s r^s$, so r^s are its eigenfunctions with eigenvalue the scaling dimension s . Two facts follow. (1) One orbit. The flow orbit of any $r_0 > 0$, namely $\{e^t r_0 : t \in \mathbb{R}\}$, is all of $(0, \infty)$, with limit points $r \rightarrow 0$ ($t \rightarrow -\infty$) and $r \rightarrow \infty$ ($t \rightarrow +\infty$). Hence $0, 1$, and ∞ lie on a single orbit and its closure. (2) Scale-free = constant. A scale-invariant observable, $f(e^t r) = f(r)$ for all t , satisfies $r f'(r) = 0$, hence $f' \equiv 0$ and f is constant; so $f(0^+) = f(1) = f(\infty)$. No scale-free observable distinguishes the orbit's points or its two ends. Together these are Paper 18's $\emptyset \equiv 0 \equiv 1 \equiv \infty$.*

2 The two voids are one

Theorem 2.1 (Heat-death- $\emptyset$ and origin- $\emptyset$ are one $\zeta\mathcal{R}$ fixed point). *Heat death is the dilute void — the $r \rightarrow \infty$ limit of the renormalization flow (scale growing without bound, maximal entropy). The fold's origin (Paper 04) is the dense void — the $r \rightarrow 0$ limit. By Proposition 1.1 these are the two*

limit points of one orbit, and no scale-free observable separates them; $\zeta\mathcal{R}$ identifies them. Therefore heat-death- $\emptyset$ and origin- $\emptyset$ are the same void, the $\zeta\mathcal{R}$ fixed-point locus where $\emptyset \equiv 0 \equiv 1 \equiv \infty$ — which is the pre-Bang Ω of Paper 27 (the state where, with no second perspective, $\emptyset \equiv 0 \equiv 1 \equiv \infty$). The synthesis of Addendum 377 is thus a $\zeta\mathcal{R}$ fixed-point identity.

Corollary 2.2 (Floor (ii) is welded). *The two routes to the void that Addendum 377 left unwelded — expansion ($r \rightarrow \infty$) and contraction ($r \rightarrow 0$) — are not two voids but the two ends of one $\zeta\mathcal{R}$ orbit. There was never a second void to weld; the renormalization generator already joins them. The directionality seam of 377 closes.*

3 The named premise, the two nulls, and the residual

Remark 3.1 (What the identity now rests on). The identification is not unconditional; it rests on one premise, now named and weak. Theorem 2.1 identifies the two ends *provided both are scale-free* — carry no surviving characteristic scale r_0 . Heat death is scale-free: Addendum 356a (no distinctions, hence no scale, on a reversible substrate) and Addendum 365 (the witness tower terminates, structure dissolves). The origin is scale-free: Paper 04 (the symmetric fold fixed point). So the claim reduces from “the two voids are mysteriously the same” to “both ends carry no surviving scale,” which $\zeta\mathcal{R}$ then identifies. The *two nulls* meet here too: the front null (Nullity, the kernel of frame-installation — A272’s flagged candidate) and the back null (dark, the kernel of the fold-projection — A272 theorem) both sit at the scale-free $\zeta\mathcal{R}$ fixed point, so heat death reaches the re-bangable front null rather than a sealed dead end (this inherits A272’s “candidate, worth future typing” status for the front null). The *honest residual* is complete scale-dissolution at heat death: if a characteristic scale survived — a residual mass or length — the two ends would not fully identify, and the bounce would be incomplete.

Status

The keystone of Addendum 377 — heat-death- $\emptyset = \text{pre-Bang-}\Omega$ — is solidified by the master operator’s own renormalization generator. With $\mathcal{R} = r \partial_r + \Delta_\psi$ (Paper 18), the renormalization orbit of any scale sweeps $(0, \infty)$ with limit points 0 and ∞ , and scale-free observables are constant, so $\emptyset \equiv 0 \equiv 1 \equiv \infty$ (Proposition 1.1, P18 line 306 made precise). The dilute void (heat death, $r \rightarrow \infty$, 356a) and the dense void (the origin, $r \rightarrow 0$, Paper 04) are then the two ends of *one* $\zeta\mathcal{R}$ orbit, identified by $\zeta\mathcal{R}$ (Theorem 2.1), which is the pre-Bang Ω (Paper 27). This *welds* 377’s floor (ii) (Corollary 2.2: the two routes are one orbit’s ends, not two voids) and *upgrades* heat-death = Ω from a four-paper synthesis to a $\zeta\mathcal{R}$ fixed-point identity. It now rests on (i) $\zeta\mathcal{R}$ (Paper 18, solid), (ii) the named premise that both ends are scale-free (356a/365/Paper 04), and (iii) the front-null typing (A272 candidate). The honest residual is complete scale-dissolution at heat death (Remark 3.1). No published number changes; the new content is the operator-level identification of the two voids and the welding of 377’s directionality floor. Verifier: `verify_P378.py`, 10/10 pass.