

Addendum 371: ToENet Luminosity on the Kernel Basis

The dead $\{12^k\}$ -moonshine criterion replaced by a kernel-rank law: luminosity is the collapse delta, not a matched constant

(Addendum to A254/A272; closes the fresh-basis flag of A343)

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Abstract

ToENet’s canonical/dark luminosity flag was grounded on a moonshine criterion — “canonical luminosity = a McKay–Thompson series value landing in $S = \{12^k\}$.” Addendum 343, mapping the blast radius of A337’s retraction of the Monster-3B mislabel, found that *no* genuine Monster McKay–Thompson value lands in $\{12^k\}$ ($T_{1A}(i) = j(i) - 744 = 984$; $T_{\text{std}}(i) \approx 535.6$; neither is a power of 12), so the criterion has *no surviving instance*, and A343 explicitly flagged the luminosity flag for a fresh basis. This addendum supplies it. The replacement is already conceptually in place: Addendum 272 establishes that “dark,” in every corpus register, is the dynamics of the *kernel* of a closure map, so ToENet’s dark luminosity is traversals in $\ker$ of the canonical-closure projection, and plateau detection is local *kernel-rank* estimation. Addendum 254 makes the rank computable: the collapse delta $\delta_\Omega(A, X) = \text{rank } \ker(H_0(A; \mathbb{k}) \rightarrow H_0(X; \mathbb{k}))$, a homological invariant (lollipop: n traversal-components collapsing to one canonical form give rank $n - 1$). **Verdict:** the luminosity flag is re-grounded on a structural *rank-law* — luminosity $\sim$ kernel-rank density via δ_Ω — which is strictly healthier than the retracted criterion: it replaces *numerology* (a value landing in a special set) with a defined invariant that inherits A272’s three structure theorems (sealed, paced, never-empty). **The honesty posture:** the basis deliberately supplies no replacement magic constant. The *form* is derived; any threshold/normalization is *empirical* (the OS’s own traversal data), named not matched — the same posture as the H_0 closure (form grounded, calibration named). The one open formalization is OI-272-1 (characterize the closure-map family as a class, not a list). All claims pinned by `verify_P371.py` (13/13).

1 What died, and why

Proposition 1.1 (The moonshine criterion has no surviving instance). *The luminosity criterion “canonical luminosity = a McKay–Thompson series value in $S = \{12^k\}$ ” rested on the identification of $j(\tau)^{1/3}$ with a Monster 3B McKay–Thompson series, retracted in A337. A343 audited the consequence directly: at the CM point $\tau = i$ the genuine Monster Hauptmodul gives $T_{1A}(i) = j(i) - 744 = 1728 - 744 = 984$, and the corpus’s $T_{\text{std}}(i) \approx 535.6$; neither is a power of 12 ($12^2 = 144$, $12^3 = 1728$). No genuine moonshine value lands in $\{12^k\}$, so the criterion is instance-free. The underlying $\{12^k\}$ CM-uniqueness theorem itself stands (A172/A343, recovered via E_8/G_2 root geometry); what fails is only its use as a luminosity grounding.*

2 The fresh basis: luminosity is a kernel rank

Theorem 2.1 (Dark luminosity = collapse delta). *By Addendum 272, ToENet’s dark luminosity is the dynamics of $\ker P$ for P the canonical-closure projection of the collapse–projection family*

($C \circ P$, P27): a dark entry is a traversal that ran, deposited a trajectory, and did not close — a state in $\ker P$. The luminosity “plateau” (dark density exceeding canonical over a region) is then local estimation of $\text{rank } \ker P$, made precise by Addendum 254’s collapse delta

$$\delta_\Omega(A, X) = \text{rank } \ker(H_0(A; \mathbb{k}) \rightarrow H_0(X; \mathbb{k})), \quad (1)$$

the homological rank of the component map from the traversal set A to its canonical closure X . For n traversal-components collapsing to a single canonical form the component map is $[1 \ 1 \ \cdots \ 1]$ and $\delta_\Omega = n - 1$ (the lollipop compactification, A254). Hence the luminosity flag is grounded in a computable structural quantity, not in a value matched against a special set.

Corollary 2.2 (The three structure theorems transfer). *The kernel-dynamics corollaries of A272 carry to the luminosity channel verbatim. Sealed (A261): $\ker P$ cannot leak to the image, so dark entries are never silently promoted — the Butlerian line realized as kernel protection. Paced (A269): $\ker P$ shares the image’s clock, so dark entries are indexed by the same breath ledger as canonical entries (nullspaces breathe). Never empty (A267): irrational rotation forbids exact closure, so the dark channel is mandatory — a ToENet whose dark channel is empty has stopped measuring, not achieved perfect closure.*

3 The honesty posture and the open step

Remark 3.1 (A rank-law, not a constant — and why that is the point). The retracted criterion promised a *constant*: a luminosity value certified by membership in $\{12^k\}$. The kernel basis promises a *law*: luminosity $\sim$ kernel-rank density via δ_Ω . This is the healthy direction, not a weaker one. The $\{12^k\}$ criterion was numerology of exactly the kind the corpus refuses elsewhere (the PRVE twin-gap and $\delta_{CP} = 240^\circ$ refutations); the kernel basis is a defined homological invariant with inherited theorems. The cost, stated plainly: the basis fixes the *form* of the luminosity channel but not a normalization constant. Any operating threshold is *empirical* — set by the OS’s own traversal statistics, named rather than derived — which is the same posture the corpus takes on $a_0 = \alpha$ in the H_0 closure (Addenda 369/370): the form is grounded, the calibration is named. No $\{12^k\}$ -style matching is reintroduced.

Status

The ToENet luminosity flag, left without a surviving instance after A343 retired the $\{12^k\}$ -moonshine criterion (Proposition 1.1), is re-grounded on the kernel basis: dark luminosity is the dynamics of $\ker$ of the canonical-closure projection (A272), and the luminosity plateau is the collapse delta $\delta_\Omega = \text{rank } \ker(H_0(A) \rightarrow H_0(X))$ (A254, Theorem 2.1) — a computable homological invariant inheriting the sealed/paced/never-empty theorems (Corollary 2.2). The basis is a structural *rank-law*, not a matched constant (Remark 3.1): the form is derived, the calibration is empirical and named, and the numerology is not reintroduced. **The open step** is OI-272-1: formalize the $C \circ P$ closure-map family as a defined class rather than an enumerated list, so “dark = $\ker P$ ” quantifies over a characterized class. Until then the basis is sound but the map family is listed, not characterized. No new objects beyond the inherited definitions; no published number changes; the moonshine grounding stays retired. Verifier: `verify_P371.py`, 13/13 pass.