

Addendum 367: Xavier–Stokes Against the Discovered Unstable Singularities

A gradient-power-counting test of the $1/\pi$ strain back-pressure
versus the machine-found self-similar blow-ups (arXiv:2509.14185)
(Addendum to Paper 22)

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Abstract

Wang, Buckmaster, Lai et al. (arXiv:2509.14185, 2025) systematically discovered families of *unstable* self-similar blow-up solutions of the Euler/IPM/Boussinesq equations, indexed by a blow-up rate λ and an order of instability n (e.g. $\lambda_n \sim 1/(1.4187n + 1.0863) + 1$ for Boussinesq/Euler), with the decisive observation that the most-unstable (small- λ) singularities are the ones expected to overcome dissipation and persist from Euler to viscous Navier–Stokes. Paper 22 (Xavier–Stokes) adds an edge-layer strain back-pressure $\nu_{\text{eff}} = \nu(1 + \psi/\pi)$ with $\psi \sim \beta\tau|S|^2$ and the $1/\pi$ fixed by the edge/boundary ratio of $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$. This addendum tests whether that back-pressure defeats the discovered singularities, by gradient power-counting near the blow-up ($G = \text{velocity-gradient scale} \rightarrow \infty$, $\ell = \text{length scale} \rightarrow 0$). **Result.** The three competing terms scale as: vortex stretching $\omega\omega S \sim G^3$ (three gradient powers); standard viscosity $\nu|\nabla\omega|^2 \sim \nu G^2/\ell^2$; and the Xavier–Stokes enhanced dissipation $\nu(\psi/\pi)|\nabla\omega|^2 \sim (\nu/\pi)G^4/\ell^2$, since $\psi \sim |S|^2 \sim G^2$ adds *two* gradient powers. Hence

$$\frac{\text{enhanced}}{\text{stretching}} \sim \frac{\nu}{\pi} \frac{G}{\ell^2} \longrightarrow +\infty$$

always, independent of λ : the $1/\pi$ back-pressure defeats the entire discovered hierarchy. The margin (exponent $p + 2q > 0$ for a model $G \sim (T - t)^{-p}$, $\ell \sim (T - t)^q$) *grows* with severity, so the most-unstable singularities are the *easiest* to defeat — a clean, counter-intuitive, falsifiable prediction, and the scaling-level explanation of Paper 22’s global-regularity theorem. **Honest limits.** This is the mechanism of Xavier–Stokes, whose enhanced dissipation has four gradient powers; *real* Navier–Stokes has the two-power dissipation $\nu G^2/\ell^2$, which beats the three-power stretching only for some λ (λ -dependent) — exactly why NS blow-up is open and the discovered singularities matter. So the test confirms the regularization and quantifies its mechanism; it does not resolve NS. And the $1/\pi$ sets only the *onset* of the back-pressure, not the asymptotic dominance, so it is not constrained by the test. All claims pinned by `verify_P367.py` (10/10).

1 The test: power-counting near the singularity

Proposition 1.1 (Three scalings). *Near a self-similar blow-up with gradient scale $G \rightarrow \infty$ and length scale $\ell \rightarrow 0$, the three terms governing enstrophy growth (and the Beale–Kato–Majda integral) scale as*

$$\underbrace{\omega_i \omega_j S_{ij}}_{\text{stretching}} \sim G^3, \quad \underbrace{\nu |\nabla \omega|^2}_{\text{standard}} \sim \frac{\nu G^2}{\ell^2}, \quad \underbrace{\nu \frac{\psi}{\pi} |\nabla \omega|^2}_{\text{Xavier–Stokes}} \sim \frac{\nu}{\pi} \frac{G^4}{\ell^2},$$

the last because the structure field $\psi \sim \beta\tau|S|^2 \sim G^2$ contributes two extra gradient powers, giving four in the enhanced dissipation against the stretching's three.

Theorem 1.2 (The back-pressure defeats every discovered singularity). *From Proposition 1.1, enhanced/stretching $\sim (\nu/\pi)G/\ell^2 \rightarrow +\infty$ as $G \rightarrow \infty$, $\ell \rightarrow 0$, for every admissible blow-up rate λ and order of instability n . In the model $G \sim (T-t)^{-p}$, $\ell \sim (T-t)^q$ ($p, q > 0$), the enhanced term's exponent exceeds the stretching's by $p + 2q > 0$; this margin is positive for all p, q and increases with the severity p . Hence the Xavier–Stokes enhanced dissipation outgrows the vortex stretching near every self-similar singularity, defeating the entire discovered hierarchy, and defeating the most-unstable (most severe) members by the largest margin.*

Remark 1.3 (The counter-intuitive prediction, and Paper 22's theorem). The members of the discovered hierarchy that “overcome the most dissipation” (the small- λ , high- G ones, which survive even standard viscosity) are precisely the ones the strain back-pressure defeats most easily, because $\psi \sim G^2$ grows fastest exactly where the gradient is steepest. This is the scaling-level content of Paper 22's global-regularity theorem: the four-gradient-power dissipation cannot be outrun by three-gradient-power stretching.

2 What the test does not show

Remark 2.1 (Real NS is λ -dependent and open). The Xavier–Stokes win is a property of its *four*-power dissipation. Real Navier–Stokes has the *two*-power dissipation $\nu G^2/\ell^2$, and standard/stretching $\sim \nu/(G\ell^2)$ has a λ -dependent sign: some singularities (large p) survive standard viscosity, others do not. This λ -dependence is exactly the open Navier–Stokes blow-up problem, and is why the discovered unstable singularities are significant. The test therefore confirms the Xavier–Stokes regularization is robust and explains its mechanism; it does *not* resolve Navier–Stokes (Paper 22's stated caveat).

Remark 2.2 (The $1/\pi$ is onset, not dominance; the physical crux). The dominance in Theorem 1.2 is exponent-driven ($G/\ell^2 \rightarrow \infty$) and therefore robust to the value of the coupling: $1/\pi$ sets only the *onset* of the back-pressure (where $\psi/\pi \sim 1$, i.e. $|S|^2 \sim \pi/(\beta\tau)$), not whether it wins. So this test does not constrain $1/\pi$ — aligning the geometric coupling with the discovered λ_n coefficients would be a post-hoc fit, the kind the corpus discipline refuses. The one genuine open question the discovered singularities sharpen is physical: whether the edge-layer ψ -coupling is present in real fluids. If it is, the discovered singularities are artifacts of dropping the edge layer; if it is not, Xavier–Stokes is a geometric regularization and the Millennium problem stands.

Status

The $1/\pi$ strain back-pressure of Paper 22 defeats the entire family of unstable self-similar singularities discovered in arXiv:2509.14185. Near any such blow-up the Xavier–Stokes enhanced dissipation scales as $(\nu/\pi)G^4/\ell^2$ — the strain field $\psi \sim |S|^2 \sim G^2$ adds two gradient powers — and outgrows the vortex stretching G^3 by the factor $G/\ell^2 \rightarrow \infty$, for every blow-up rate and order of instability, with the margin growing in severity (Theorem 1.2). This is the scaling-level explanation of Paper 22's global-regularity theorem and yields the falsifiable, counter-intuitive prediction that the most-unstable singularities are the easiest for the back-pressure to defeat (Remark 1.3).

The test confirms the regularization, not a resolution of NS: the win uses the four-power Xavier–Stokes dissipation, whereas real NS has the two-power dissipation whose contest with the three-power stretching is λ -dependent and open (Remark 2.1); and $1/\pi$ sets the onset, not the

dominance, so the test does not constrain it (Remark 2.2). The concordance with the discovered work is structural and strong — both center the strain rate $|S|^2$ — and it equips Paper 22 with a concrete catalog of singularities its back-pressure provably defeats. The crux it sharpens is the physical reality of the edge layer. No published number changes. Verifier: `verify_P367.py`, 10/10 pass.

Data source. Y. Wang, M. Bennani, J. Martens, ..., T. Buckmaster, J. Gómez-Serrano, C.-Y. Lai, “Discovery of Unstable Singularities,” arXiv:2509.14185 [math.AP], 2025.