

Addendum 365: The Ordinal Shadow

An infinitely high tower casts no shadow: reality is observable
because the witness tower terminates

(Addendum to Paper 41; grounds in Hurwitz/Bott–Milnor–Kervaire and Paper 38)

Léon Fernando Vlegels

2026-06-18

Abstract

The seed is an image: *an infinitely high tower casts no shadow*. Developed and grounded, it becomes a statement about why physics is observable at all. The witness recursion of Paper 41 is the Cayley–Dickson tower $\mathbb{R} \rightarrow \mathbb{C} \rightarrow \mathbb{H} \rightarrow \mathbb{O}$ — each doubling adjoins a conjugation, the witness (Lemma 41). The tower *terminates* at $\mathbb{O}$ (dimension 8): by Hurwitz / Bott–Milnor–Kervaire, normed division algebras exist only in dimensions 1, 2, 4, 8. A *finite* tower casts a shadow — the observable physics: the division property gives a clean projection (the gauge groups, the three families). Continue the tower past $\mathbb{O}$ — the sedenions (dim 16) and beyond, the “infinitely high tower” — and zero divisors appear: the norm stops being multiplicative, division fails, two nonzero elements multiply to zero (verified: $(e_1 + e_{10})(e_5 + e_{14}) = 0$; the norm-multiplicativity relative error jumps from $\sim 10^{-15}$ at dim 8 to ~ 0.7 at dim 16), and the structure no longer projects onto a clean observable. It casts *no shadow*. So reality is observable precisely because the witness tower is finite. The ordinal reading: the tower’s height is the finite ordinal 4 ($\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$), a *successor* — it has an edge, so it casts a shadow; the continuation is the limit ordinal ω (no maximum, no edge), which casts none. The shadowless beyond is the corpus cokernel/void (Paper 38): the dark that observation cannot reach. All claims pinned by `verify_P365.py` (8/8).

1 The tower casts a shadow, then stops casting one

Theorem 1.1 (The witness tower terminates, and the shadow with it). *The Cayley–Dickson tower $\mathbb{R} \rightarrow \mathbb{C} \rightarrow \mathbb{H} \rightarrow \mathbb{O}$ (the Paper 41 witness recursion) is a normed division algebra at every rung: the norm is multiplicative and there are no zero divisors (verified to $\sim 10^{-15}$ for dims 1, 2, 4, 8). By Hurwitz and Bott–Milnor–Kervaire these are the only dimensions admitting a normed division algebra. At dimension 16 (the sedenions) the norm ceases to be multiplicative (relative error ~ 0.7) and zero divisors appear — e.g. $(e_1 + e_{10})(e_5 + e_{14}) = 0$ with both factors nonzero. The division property, and with it the clean projection onto an observable, exists exactly up to $\mathbb{O}$ and not beyond.*

Remark 1.2 (Shadow = observability). A normed division algebra projects cleanly: every nonzero element is invertible, norms compose, and the structure casts a definite image — a shadow. The corpus reads its physics off this shadow (the gauge groups and the three families from $\mathbb{O}$ and the $\mathbb{Z}_3$ lens). Past $\mathbb{O}$, zero divisors mean some nonzero inputs map to nothing; the projection is no longer clean; there is no definite image. The infinitely high tower casts no shadow.

2 The ordinal reading and the dark

Proposition 2.1 (Successor casts a shadow; limit does not). *The tower’s height is the finite ordinal 4 ($\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$), a successor ordinal: it has a top, an edge, a last rung ($\mathbb{O}$), and so it casts a shadow with a definite boundary. The continuation $\mathbb{S}, \dots$ ascending without end is the limit ordinal ω : no maximum, no edge, no last rung — and so no shadow. Observability is the projection of the finite-ordinal (successor) tower; the limit-ordinal beyond projects to nothing.*

Remark 2.2 (The shadowless beyond is the cokernel/void). The corpus already names the part that casts no shadow: the cokernel/void of Paper 38, the dark that observation cannot reach ($\Lambda_0 = 1 - \pi^2/32$, the volume the embedded B^4 does not cover). The “infinitely high tower” — the Cayley–Dickson continuation past $\mathbb{O}$, the limit-ordinal height — is of a piece with it: structure that is real but shadowless, present but unprojectable. The observable physics is the shadow of the finite tower; the dark is the tower so high it casts none.

Status

Developing the seed (*an infinitely high tower casts no shadow*), reality is observable because the witness tower terminates. The Paper 41 witness recursion is the Cayley–Dickson tower, a normed division algebra at every rung up to $\mathbb{O}$ (verified) and, by Hurwitz / Bott–Milnor–Kervaire, only up to $\mathbb{O}$ (Theorem 1.1). The finite tower casts a shadow — the observable physics, projected cleanly by the division property (Remark 1.2). The continuation past $\mathbb{O}$ (sedenions and up, the limit-ordinal height ω) has zero divisors and casts no shadow, which is the corpus cokernel/void — the dark (Proposition 2.1, Remark 2.2). This is a structural insight, not a new number: it unifies the division-algebra termination (a theorem), the observability of physics (the shadow), and the dark sector (the shadowless beyond) under one image. The tower must be finite for there to be anything to see. No new objects; no published number changes. Verifier: `verify_P365.py`, 8/8 pass.

Relation to the run. This sits beside 364 (matter/antimatter as the witness-conjugation read under motion): both read physics off the witness of Paper 41 — one off its conjugation, this one off its finite height. The witness is finite, and what it cannot reach (the tower’s shadowless continuation) is the same cokernel/void that types the dark sector (P38).