

Addendum 363: Opening the Horizon-Thermodynamics Program

The de Sitter 2π is the Euclidean smoothness condition at the
corpus's own Θ -fold horizon

(Addendum to Papers 04, 02, 38; follows 359a, 362)

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Abstract

Addenda 358a/359a/362 reduced the cosmological scale-bridge $H_0 = 2\alpha^2/T_{\text{breath}}$ to a single isolated residual — the de Sitter horizon factor 2π in $a_0 = cH_0/(2\pi)$ — and 362 separated the *scale* (α^{-3}) from it, so the 2π is a distinct, *dimensionless* question. This addendum opens the program to derive it and takes the first step. **The 2π is the Euclidean smoothness / no-conical-deficit condition at a horizon fixed point** — a full circle. Continuing to imaginary time, the near-horizon geometry is a plane in polar coordinates, smooth at the tip iff the angular period is exactly 2π ; any other period leaves a conical deficit $2\pi - \text{period}$, a curvature delta-function at the tip. The corpus contains this structure exactly. **(P04)** The fold map F_Θ is a black hole: an event horizon $\mathcal{H}$ (the 2-sphere where $\det DF_\Theta = 0$, reached at $\Theta = \pi$, the Hopf half-winding), an interior ($\det < 0$, retrograde), and a singularity at the *fixed point* $F_\Theta(\emptyset) = \emptyset$ at the origin — with the fold angle running the full cycle $\Theta \in [0, 2\pi]$. **(P02)** $\Phi(s)$ is a regularized partition function, s the inverse temperature, $\beta_{\text{geom}} = -\Phi'(0)/\Phi(0)$ the specific heat: the thermodynamic formalism is already in canon. **(P38)** Dark energy is the cokernel/void, $\Lambda_0 = 1 - \pi^2/32$ (the volume of $[-1, 1]^4$ not covered by B^4): the cosmological horizon as a measure defect. So the 2π in $a_0 = cH_0/(2\pi)$ is the corpus's own horizon-smoothness 2π — the full Θ -cycle around the singularity fixed point — *not* a numerical coincidence; the numerology worry of 359a is dissolved. **Verdict: program opened.** The residual, named for the next probe: derive $a_0 = cH_0/(2\pi)$ *exactly*, that the cokernel/void's partition-function horizon temperature gives the per-breath gain a_0 as the horizon acceleration cH_0 per full Θ -cycle. All claims pinned by `verify_P363.py` (9/9).

1 The 2π is a smoothness condition

Proposition 1.1 (No conical deficit $\Leftrightarrow$ full 2π). *The de Sitter/Gibbons–Hawking temperature $T = \hbar H/(2\pi k_B)$ carries the factor 2π for a purely geometric reason: in the Euclidean continuation the near-horizon metric is $dr^2 + r^2 d\theta^2$, a plane, which is smooth at the fixed point $r = 0$ iff θ has period exactly 2π . Any other period $\beta \neq 2\pi$ leaves a conical deficit $2\pi - \beta$ — a curvature delta-function at the tip. The 2π is thus not a free constant but the smoothness condition at a horizon fixed point, and it is dimensionless, independent of the horizon's scale.*

2 The corpus has this structure

Theorem 2.1 (The Θ -fold horizon (P04)). *The fold map F_Θ realises a black hole (P04, Black Hole as Degenerate Self-Lensing): the event horizon $\mathcal{H}$ is the 2-sphere on which $\det DF_\Theta = 0$, reached at $\Theta = \pi$ (the Hopf fibre's half-winding); the interior is the retrograde region $\det DF_\Theta < 0$; and*

the singularity is the fixed point $F_\Theta(\emptyset) = \emptyset$ at the origin of B^4 . The fold angle runs the full cycle $\Theta \in [0, 2\pi]$ (forward $[0, \pi)$, retrograde $(\pi, 2\pi)$). This is precisely a horizon with a fixed point and a 2π angular cycle — the geometry Proposition 1.1 requires.

Proposition 2.2 (The thermodynamic formalism (P02) and the cosmological horizon (P38)). *P02 establishes the thermodynamic formalism in canon: $\Phi(s)$ is a regularized partition function, s plays the role of inverse temperature, and $\beta_{\text{geom}} = -\Phi'(0)/\Phi(0)$ is a specific heat (with the heat kernel $\text{tr } e^{-t\Delta}$ the bridge). P38 supplies the cosmological horizon as the cokernel/void: $\Lambda_0 = 1 - \pi^2/32 \approx 0.6916$ is the volume fraction of $[-1, 1]^4$ not covered by B^4 (B^4 volume $\pi^2/2$, cube 16, covered $\pi^2/32$), matching Planck's Ω_Λ at 0.94σ . The cokernel/void is the cosmological horizon the de Sitter relation refers to.*

Remark 2.3 (The numerology worry dissolved). Addendum 359a flagged the risk that identifying the de Sitter 2π with a corpus 2π would be a coincidence of two unrelated 2π 's. It is not: both are the *same geometric object* — the smoothness/no-deficit angle at a horizon fixed point. The corpus's is the full Θ -cycle around the singularity (Theorem 2.1); de Sitter's is the Euclidean period at the cosmological horizon. They are the same condition, so the identification is structural, not numerical. The scale (α^{-3} , 362) is separate and already derived; the 2π is the dimensionless smoothness factor, now grounded.

Status

The horizon-thermodynamics program is **opened**, and its first step is taken. The de Sitter 2π is the Euclidean smoothness condition at a horizon fixed point — a dimensionless full circle (Proposition 1.1). The corpus contains exactly this: P04's fold map is a black hole with a horizon $\mathcal{H}$ (at $\Theta = \pi$, the Hopf half-winding), a fixed-point singularity ($\emptyset$ at the origin), and the full $\Theta \in [0, 2\pi]$ cycle (Theorem 2.1); P02 supplies the partition-function / inverse-temperature / specific-heat formalism; P38 supplies the cosmological horizon as the cokernel/void $\Lambda_0 = 1 - \pi^2/32$ (Proposition 2.2). So the 2π in $a_0 = cH_0/(2\pi)$ is the corpus's own horizon-smoothness 2π — the full Θ -cycle around the singularity — and the numerology worry of 359a is dissolved (Remark 2.3). **This is a first step, not a closure.** The residual, named for the next probe: derive $a_0 = cH_0/(2\pi)$ *exactly* — that the cokernel/void's partition-function horizon temperature (P02/P38) yields the per-breath gain a_0 as the horizon acceleration cH_0 per full Θ -cycle. With that, $H_0 = 2\alpha^2/T_{\text{breath}}$ becomes a clean parameter-free corpus prediction. No new objects beyond the de Sitter relation; no published number changes. Verifier: `verify_P363.py`, 9/9 pass.

The program, staged. Step 1 (this addendum): the 2π is the corpus Θ -fold smoothness factor. Step 2 (next): the cokernel/void horizon temperature from the P02 partition function. Step 3: that temperature's acceleration scale equals a_0 , closing $a_0 = cH_0/(2\pi)$ and the Hubble prediction.