

Addendum 360: The Prime Number Theorem and Goldbach in TOE Context

PNT as the leading pole of the renormalisation face; the Goldbach
2-vs-3 parity as dimension, not metric
(Addendum to Paper 24)

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Abstract

Two number-theory statements read through the master operator. **Part A (solid).** The prime number theorem $\pi(x) \sim x/\ln x$ is the leading-order spectral shadow of the renormalisation face $\zeta\mathcal{R} = r\partial_r$ of $\hat{O}$. The dilation operator $x\partial_x$ has Mellin eigenfunctions x^s (eigenvalue s); Paper 24's $H = -i(x\partial_x + \frac{1}{2})$ is its self-adjoint symmetrisation, whose real spectrum is the critical line $\text{Re}(s) = \frac{1}{2}$ (the Riemann boundary spectrum). PNT is the weaker statement that ζ has no zero on $\text{Re}(s) = 1$; its main term $\pi(x) \sim \text{Li}(x) \sim x/\ln x$ is the $s = 1$ pole of ζ (residue 1) read back through the inverse Mellin transform. So the $1/\ln x$ prime density is the leading pole of the $\zeta\mathcal{R}$ face — a corpus identification, not an analogy (verified: $\pi(x)/(x/\ln x) \rightarrow 1$, $\pi(x)/\text{Li}(x) \rightarrow 1$). **Part B (a bounded resonance).** The binary (strong, even $n = p + q$) and ternary (weak, odd $n = p + q + r$) Goldbach conjectures — ternary proven (Helfgott 2013), binary open — map onto the TOE by *parity*, not metric. The Hardy–Littlewood count is $r_k(n) \sim \mathfrak{S}_k(n) n^{k-1}/(\ln n)^k$: the power of $\ln$ equals the number of primes k (one PNT density per prime, on the $(k-1)$ -simplex), so binary gives $(\ln n)^2$ and ternary $(\ln n)^3$ (verified). The reason ternary closes and binary is marginal — the $k = 3$ main term dominates the minor-arc error, $k = 2$ does not — is the same dimensional fact as the corpus's “composition closes at three” ($\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ is a three-term sum; the $\mathbb{Z}_3$ lens, the three layers) versus the two-fold dual-observer (Papers 27/28) that keeps an irreducible residual. **But the layer weights $4\pi^3:\pi^2:\pi$ do not enter the representation counts:** $\mathfrak{S}_k(n)$ is the arithmetic singular series $\prod_{p|n}(\cdots)$, set by n 's prime divisors, not by any π -power (verified: $r_2(99990) = 1855 \gg r_2(100000) = 810$, $r_2(99991) = 1$). The connection is dimensional, not metric — explicitly *not* a claim that the TOE proves or quantitatively predicts Goldbach. All claims pinned by `verify_P360.py` (11/11).

1 Part A: PNT is the leading pole of the renormalisation face

Proposition 1.1 (The dilation operator and the $s = 1$ pole). *The renormalisation face of the master operator is $\zeta\mathcal{R} = r\partial_r$ (Paper 18; the dilation generator). On the multiplicative half-line it is $x\partial_x$, with eigenfunctions x^s and eigenvalue s : the Mellin transform diagonalises it. Paper 24 takes its self-adjoint symmetrisation $H = -i(x\partial_x + \frac{1}{2})$, whose real spectrum forces $\text{Re}(s) = \frac{1}{2}$ — the Riemann critical line (the boundary spectrum). The prime number theorem is the weaker statement that ζ has no zero on $\text{Re}(s) = 1$, equivalently that the inverse Mellin transform of ζ'/ζ is dominated by the simple pole at $s = 1$ (residue 1), giving the main term $\pi(x) \sim \text{Li}(x) \sim x/\ln x$. Numerically $\pi(x)/(x/\ln x) \rightarrow 1$ and $\pi(x)/\text{Li}(x) \rightarrow 1$ (the latter to 5 figures by $x = 2 \times 10^6$).*

Remark 1.2 (Why this is a real identification). The $1/\ln x$ density is not imposed; it is the residue of the $s = 1$ pole transported by the $\zeta\mathcal{R}$ face. The corpus already builds this operator (Paper 24) and is careful to claim only the Hilbert–Pólya framework, not a proof of RH. The PNT statement is strictly the leading pole and is unconditional; it sits one layer below RH on the same operator.

2 Part B: the Goldbach 2-vs-3 parity is dimensional

Proposition 2.1 (The $\ln$ -exponent counts the primes). *For n a sum of k primes the Hardy–Littlewood asymptotic is $r_k(n) \sim \mathfrak{S}_k(n) n^{k-1}/(\ln n)^k$: each prime contributes one PNT density factor $1/\ln n$, and the compositions fill the $(k-1)$ -simplex (volume $\sim n^{k-1}$). Hence binary ($k = 2$) carries $(\ln n)^2$ and ternary ($k = 3$) carries $(\ln n)^3$. Verified: $r_2(n)(\ln n)^2/n$ stays ≈ 1.05 – 1.08 and $r_3(n)(\ln n)^3/n^2$ stays bounded across n . The proof status follows the same dimension: the $k = 3$ main term dominates the minor-arc error (ternary proven, Helfgott 2013), while $k = 2$ is marginal (binary open).*

Remark 2.2 (The corpus parity). The same “three closes, two leaves a residual” is the corpus’s own signature. Composition closes at three: $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ is a three-term layer sum, the $\mathbb{Z}_3$ lens $L(3,1)$ is the rigid family structure, and the three-layer ontology is complete. The two-fold structure — the dual observer of Papers 27/28, “why one reader is not enough” — carries a permanent residual (the standing wave’s minus sign). Ternary Goldbach closing and binary staying open echoes this parity exactly.

Theorem 2.3 (The limit: weights do not transfer). *The geometric layer weights $4\pi^3 : \pi^2 : \pi$ do not appear in the Goldbach representation counts. The constant $\mathfrak{S}_k(n)$ is the arithmetic singular series $\prod_{p|n}(\cdots)$, governed entirely by the small-prime divisors of n : numerically $r_2(99990) = 1855$ (with 99990 divisible by 2, 3, 5, 11) against $r_2(100000) = 810$ (divisible by 2, 5) and $r_2(99991) = 1$ (99991 prime). The Goldbach comet’s scatter is arithmetic, not geometric. Hence the TOE–Goldbach connection is one of dimension (the count k , the $(k-1)$ -simplex, the $(\ln)^k$ parity), not of metric (the π -weights).*

Status

Part A is solid. PNT is the leading-order spectral shadow of the renormalisation face $\zeta\mathcal{R} = r\partial_r$: the dilation operator whose Mellin eigenfunctions are x^s , whose self-adjoint symmetrisation (Paper 24) has the critical line as spectrum, and whose $s = 1$ pole is the $x/\ln x$ main term. This is a corpus identification, verified numerically, and sits one layer below the (unproven) RH on the same operator. **Part B is a bounded structural resonance.** The Goldbach binary/ternary split maps onto the TOE by parity: the $\ln$ -exponent equals the number of primes k (binary 2, ternary 3), the $(k-1)$ -simplex picture gives “three closes, two marginal,” and this matches the corpus’s three-fold closure ($\alpha = 4\pi^3 + \pi^2 + \pi$, $\mathbb{Z}_3$) versus the two-fold dual-observer residual. The honest limit, proven here: the geometric weights $4\pi^3 : \pi^2 : \pi$ do *not* enter the representation counts, which are fixed by the arithmetic singular series. So the connection is dimensional, not metric, and is explicitly *not* a claim that the TOE proves or quantitatively predicts Goldbach — a discipline the PRVE prime-resonance programme earned, where most family-resonance claims were retracted as artifacts. No new objects; no published number changes. Verifier: `verify_P360.py`, 11/11 pass.

What a genuine quantitative probe would require. To upgrade Part B from resonance to result, the three-layer weights would have to predict something the singular series does not —

e.g. a geometric correction to the Goldbach comet's envelope that survives a residue-matched null.
Absent such a prediction, the parity is the content, and it is named as such.