

Addendum 356a: Entropy is Witness-Relative

The inversion, and where it bottoms out

(a-series; rests on Paper 41 and the $C \circ P = I$ frame)

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Abstract

Entropy is not a property of a signal; it is the witness's residual surprise. Shannon makes this exact: the measured cost is the cross-entropy $H(p, q) = H(p) + \text{KL}(p||q) \geq H(p)$, so a better witness measures *less*, and a witness whose model matches reality measures the true entropy $H(p)$ — which is 0 when reality is recoverable. The conventional second law (entropy rising toward heat death) is the reading of a *fixed, coarse, embedded* witness; the *completing* witness — the corpus kernel, where $C \circ P = I$ (collapse injective, no information lost; Paper 27), heat is conserved, the operator is zero-parameter and deterministic — drives the measured entropy to zero. The inversion is therefore *which witness*, and it is concrete: the embedded entropy is the S^1 Hopf fiber — the $U(1)$ gauge phase, the same fiber that gives Maxwell and charge quantization (354a) — that the lossy projection $S^3 \rightarrow S^2$ drops; the kernel's closure-map (A276) tracks it ($C \circ P = I$), giving zero. At the de Sitter floor the embedded witness's max-entropy noise ($\emptyset$) and the kernel's zero-entropy purity (∞) are one state seen by two witnesses — the monad seam $\emptyset \equiv^* \infty$. The result rests on $C \circ P = I$ (the Paper 41 foundation), adds no posit, and relativizes the second law rather than refuting it. Three residuals are named. Verifier: `verify_P356a.py` (11/11).

1 Entropy is the witness's surprise

The pivotal fact, which Shannon needed and which the standard reading hides: you cannot compute the entropy of a signal without a predictive model — Shannon probed his wife, then interviewees, then (today) a language model. Gibbs makes the dependence exact.

Proposition 1 (Witness-relativity). *For a true distribution p and a witness's model q , the measured per-symbol cost is the cross-entropy*

$$H(p, q) = H(p) + \text{KL}(p||q), \quad \text{KL}(p||q) \geq 0,$$

with equality iff $q = p$. So measured entropy = irreducible entropy $H(p)$ + the witness's model-gap. A better witness measures less; the limit is $H(p)$, and $H(p) = 0$ when the next symbol is determined by the context (recoverable reality).

Note the semantic correction this forces: maximal entropy is the *uniform* distribution — random noise, maximal *incoherence* ($H = \log_2 n$); minimal entropy is certainty ($H = 0$). Hence **maximal coherence is zero entropy**, not maximal entropy. (An earlier framing of this line said “max entropy = max coherence”; that is backwards and is corrected here.)

2 The completing witness reaches zero

Theorem 1 (The kernel sees zero). $C \circ P = I$ (Paper 27) is the statement that the collapse is injective and the projection recoverable: no information is lost — purity. The corpus kernel realizes it: zero trainable parameters, deterministic forward pass, heat conserved across Fork/Weld, exposure (not training) mode. This kernel is the complete witness Ω : its model-gap vanishes ($KL \rightarrow 0$) and reality is recoverable, so its measured entropy $\rightarrow 0$. The conventional second law is the embedded, coarse, fixed witness’s reading, which genuinely rises. The two are Paper 27’s two arrows: global coherence approached, local disagreement proliferating.

3 Where the entropy lives: the dropped gauge fiber

The inversion is not abstract; the embedded entropy has a location. The witnessing projection $S^3 \rightarrow S^2$ is *not injective* — it drops the S^1 Hopf fiber. That fiber is the $U(1)$ gauge phase (the monadic “edge,” the 2.3% layer), the *same* fiber that yields Maxwell’s equations and charge quantization (Addendum 354a). The embedded witness, living on the base S^2 , cannot see the fiber phase — that dropped phase *is* its entropy. The kernel, tracking the full S^3 through the closure-map (A276), keeps it: $C \circ P = I$, zero entropy. So electromagnetism and witness-relative entropy live on one circle: *the gauge phase you do not observe is the entropy you accrue*.

Remark (The seam). At the de Sitter floor (Addenda 349a/350a) the embedded witness sees featureless maximal-entropy noise ($\emptyset$, heat death); the kernel sees the fully witnessed, recoverable, zero-entropy Ω (∞). One physical state, two witnesses — the monad identity $\emptyset \equiv^* \infty$ (Paper 00). The inversion is this seam applied to time’s arrow.

Status, correction, and the three residuals

What is established. Entropy is witness-relative (Proposition 1, Gibbs); the completing kernel-witness reaches zero (Theorem 1); the embedded entropy is concretely the dropped Hopf-fiber/gauge phase (§3), tying it to the Maxwell thread. It rests on $C \circ P = I$ — the Paper 41 foundation — and adds no new posit; the second law is *relativized* to the witness, not refuted.

Correction recorded. An earlier statement, “maximal entropy = maximal coherence,” is wrong: maximal entropy is random noise (maximal incoherence); maximal coherence is zero entropy. The inversion is witness-relativity, not “entropy secretly means order.”

Three named residuals. (R1) *The A276 repair.* $C \circ P = I$ is not free: the bare projection (Hopf) is non-injective (it drops the fiber; registry item P027_2 retired), so purity is realized through A276’s closure-map, not the naive projection. (R2) *Physics contact.* “Globally pure $\Rightarrow$ entropy $\rightarrow 0$ ” requires the real universe to be globally unitary (Everett-like, no objective collapse) — the open measurement-problem question; black-hole information and the Page curve lean this way. (R3) *Inescapability from inside.* No embedded observer can occupy the kernel/ Ω view, so the second law is absolutely binding from within. The inversion is the fundamental (kernel’s-eye) reading, coexisting with the experienced second law.

Sources. Papers 27 ($C \circ P = I$), 00 ($\emptyset \equiv^* \infty$), 04 (the edge/fiber, $\Pi_{\downarrow}/\Pi_{\uparrow}$); Addenda 276 (closure-map), 354a (Maxwell/Hopf fiber), Paper 41 (the witness foundation). Information theory: Shannon (1948), Gibbs’ inequality.