

Addendum 352a: The Orthogonality of Witness

Pinning the 90° step of the bootstrap

(a-series; sharpens Addendum 351a)

Léon Fernando Vlegels

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Abstract

Addendum 351a left exactly one load-bearing *interpretive* posit: that the witness of a distinction sits at 90° to it (the step that carries orthogonality, and with it the four-axis monad structure). This note pins it. In any inner-product space, two states are *perfectly distinguishable* if and only if they are *orthogonal* — the Helstrom two-state discrimination bound, $P_{\max} = \frac{1}{2}(1 + \sin \theta)$, equals 1 only at $\theta = 90^\circ$. A *clean* distinction (one side never confused with the other) therefore *forces* orthogonality; the 90° is not a drawn right angle but the distinguishability condition itself. The only remaining input — that distinctions live in an inner-product space — is the corpus’s own coherence geometry (Paper 27’s S^3 , the operator Hilbert space, projective measurement), not a fresh assumption. So $90^\circ = \text{witness}$ is downgraded from interpretive posit to *analytic fact on a canonical base*; the foundational seam relocates to “why an inner-product structure,” which is the Paper 27 witness-necessity it already rests on. Verifier: `verify_P352a.py` (8/8).

1 Distinguishability is orthogonality

A witness must tell A from $\neg A$. Ask how well it can, in an inner-product space. For two equiprobable pure states with overlap $|\langle A | \neg A \rangle| = \cos \theta$, the optimal single-shot discrimination success is the Helstrom bound

$$P_{\max}(\theta) = \frac{1}{2}(1 + \sin \theta).$$

Proposition 1 (A clean distinction is a right angle). $P_{\max}(\theta) = 1$ (the two sides are never confused) holds iff $\sin \theta = 1$, i.e. $\theta = 90^\circ$, i.e. $\langle A | \neg A \rangle = 0$. For $\theta < 90^\circ$ the overlap is nonzero and the confusion probability $1 - P_{\max} > 0$. Hence a clean distinction — one whose two sides are perfectly witnessable — requires its sides orthogonal. The 90° is the distinguishability condition, not an added geometric assumption.

This replaces 351a’s interpretive step. There, “the witness sits orthogonal” was asserted as the load-bearing posit; here it is the content of Proposition 1: to witness cleanly *is* to read an orthogonal pair.

2 The inner product is canonical, not new

Proposition 1 needs one structure: an inner product (a notion of overlap, hence of distinguishability). The corpus already supplies it, and not as a new posit:

- S^3 is the unique space supporting coherent multi-observer agreement (Paper 27) — agreement *is* an overlap structure;

- the master operator $\hat{O}$ acts on a Hilbert space (Paper 18);
- measurement is the orthogonal projection $\Pi_{\downarrow}$ (Paper 4).

So $90^\circ = \text{witness}$ reduces to “a clean distinction in the corpus’s own Hilbert geometry” = distinguishability (analytic, §1) + inner product (canonical). The step is pinned.

3 The four-axis closure, now derived not drawn

351a obtained the four monad axes from $4 \times 90^\circ$ closing a wedge — a drawn angle. With Proposition 1 the same structure is forced algebraically: n *mutually*-clean distinctions are n *mutually* orthogonal directions ($\langle e_i | e_j \rangle = 0, i \neq j$). The monad map’s right angles are the pairwise distinguishability of its axes, not a feature of the drawing (which was exactly the projection-fidelity point that started this).

The *count* four is not a separate posit either; it is the same argument continued one step. Require the distinctions to *compose without collapse* — combining two genuine distinctions never yields nothing, the no-zero-divisor form of the coherence identity $C \circ P = I$. Orthogonal directions that compose without zero divisors are precisely a *normed division algebra*, and by Hurwitz those exist only in dimensions 1, 2, 4, 8 ($\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$). The witnessing structure must moreover be associative (a consistent group of compositions) and non-abelian (order of distinction matters); the unique sphere-group meeting both is $S^3 = \text{unit quaternions} = SU(2)$ (S^7 is parallelizable but non-associative, hence not a group; S^0, S^1 are abelian). Hence the bulk is B^4 , dimension four, and the four orthogonal axes are the four quaternion components — the four scalar coefficients of $\hat{O}$, $\alpha, \gamma, \beta, \zeta$. This is the Paper 01 division-algebra derivation (Hurwitz/Adams), reached here from distinguishability rather than assumed.

Status and honest residual

What is pinned. 351a’s lone interpretive load-bearer, $90^\circ = \text{witness}$, is downgraded to an *analytic* fact (distinguishability = orthogonality, Proposition 1) resting on a *canonical* base (the Paper 27 coherence Hilbert geometry). The four-axis closure follows from mutual distinguishability rather than a drawn wedge.

What remains a seam. The residual is relocated, not removed: “why an inner-product / projective-measurement structure at all.” The answer is the Paper 27 coherence geometry — coherent agreement among witnesses requires a shared space with an overlap measure — which itself rests on the witness-necessity that opened the bootstrap. So the seam moves up to a single canonical foundation already in play, rather than sitting as a free geometric choice. No new physics; a foundation tidied.

Remark. With this, the bootstrap chain of 351a has no free interpretive load-bearer left: witness-necessity (canonical), two-sidedness (analytic), 90° (now analytic via Proposition 1), the cascade (canonical), and the ζ/Λ_0 termination (derived/canonical). The one genuine foundation is the Paper 27 inner-product coherence geometry, on which everything else now hangs analytically.