

Addendum 350b: Additive Composition

The second assembly-uniqueness sub-result: why $\hat{O}$ is a sum, not a product or an operad — P18-T2 sub-result (b)

(Addendum to Paper 18)

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Abstract

The second of the three assembly-uniqueness sub-results Paper 18 §4.4 names. Sub-result (a) (Addendum 349b) audits the components; (c) (Addenda 345b–348b) the coefficients; this addendum works (b) — why the master operator is the *sum* $\hat{O} = D_{B^4}^2 + \Delta_{S^3} + \Delta_{S^1} + V_{\text{self}} + \lambda\rho + \gamma T_{\text{cycle}} + \zeta\mathcal{R} + \beta\mathcal{M}$ rather than a product, an operadic, or any non-additive composition. The corpus-native argument has three legs. **(L1) Product geometry:** the arena is the Hopf-fibred product $(S^1 \rightarrow S^3, B^4 \text{ with boundary } S^3)$; the three kinetic sectors act on different factors, so they commute, and the canonical second-order operator on a product is the direct *sum* of factor operators, whose spectrum is the sum of factor spectra ($\Delta_{M \times N} = \Delta_M \oplus \Delta_N$) — verified on the tensor product. **(L2) Hamiltonian form:** the remaining terms enter as a potential/structure added to the kinetic part; kinetic-plus-potential is additive by the definition of a Schrödinger/Laplace-type operator, and addition is order-independent, where an operad would not be. **(L3) The additive α -decomposition:** the ground-state readout $\alpha^{-1} = \int_0^1 \rho = 4\pi^3 + \pi^2 + \pi$ is an additive layer sum (bulk $4\pi^3$ + boundary π^2 + edge π , the Paper 04 three-layer ontology); only additive composition reads it off the ground state as a sum, whereas a multiplicative composition yields a product readout ($4\pi^3 \cdot \pi^2 \cdot \pi = 4\pi^6$, or 0 when a factor vanishes), not α^{-1} . **Verdict: PARTIAL** — additive composition is shown *canonical* on these three legs; the named residual is an exhaustive exclusion of all operadic alternatives that might reproduce the same readout by another route. All claims pinned by `verify_P350b.py` (12/12).

1 The sub-result

Paper 18 §4.4 lists, among the things a uniqueness proof must establish, “that additive composition is the only admissible composition.” The canonical assembly is a sum of eight terms; the question is whether a product, an operad (an ordered composition with arity), or another non-additive law could equally satisfy the coverage requirements (R1)–(R8). We give the corpus-native case that additive composition is forced, on three independent legs, and name what the case does not deliver.

2 Leg 1: the product geometry forces additive kinetics

Proposition 2.1 (Commuting sectors, summed spectrum). *The arena is the Hopf-fibred product: B^4 with boundary S^3 and Hopf fibre S^1 ($S^1 \rightarrow S^3 \rightarrow S^2$). The three kinetic sectors $D_{B^4}^2$, Δ_{S^3} , Δ_{S^1} act on different factors, hence commute. The canonical second-order operator on a product manifold is the direct sum of the factor operators, $\Delta_{M \times N} = \Delta_M \otimes I + I \otimes \Delta_N$, whose spectrum is the sum of the factor spectra. Verified on the tensor product: the eigenvalues of the assembled kinetic*

sum equal the pairwise/triple sums of the factor eigenvalues, and the factor operators commute (simultaneous diagonalisability). The additive ground value is the sum of the sector ground values — the additive signature.

3 Leg 2: the Hamiltonian form forces additive structure terms

Proposition 3.1 (Kinetic plus potential). *The remaining terms (V_{self} , $\lambda\rho$, $\beta\mathcal{M}$, γT_{cycle} , $\zeta\mathcal{R}$) enter as a potential/structure added to the kinetic part: $\hat{O} = (\text{kinetic}) + (\text{potential/structure})$. Kinetic-plus-potential is additive by the definition of a Schrödinger/Laplace-type operator, and operator addition is commutative and associative, so the assembly order does not change $\hat{O}$. An operadic composition — an ordered law with arity — would not have this order-invariance; the corpus assembly is manifestly order-independent, which is the additive (not operadic) signature.*

4 Leg 3: the additive α -decomposition confirms it at the ground state

Theorem 4.1 (Only additive composition reads α^{-1}). *The ground-state readout is*

$$\alpha^{-1} = \int_0^1 \rho(x) dx = \underbrace{4\pi^3}_{\text{bulk}} + \underbrace{\pi^2}_{\text{boundary}} + \underbrace{\pi}_{\text{edge}} = 137.036,$$

an additive sum of the three layer integrals (the Paper 04 three-layer ontology, the $90\text{-}7\text{-}2 = 4\pi^3:\pi^2:\pi$ shell). An additive composition reads this off the ground state as a sum of layer contributions. A multiplicative composition produces a product readout: the product of the layer integrals is $4\pi^3\pi^2\pi = 4\pi^6$, not α^{-1} , and on the tensor product a multiplicative composition collapses the ground value to zero whenever a factor's ground eigenvalue vanishes. Hence only the additive composition yields the corpus α readout; the additive structure is the operator-level image of the layered geometry.

Status

Additive composition is the canonical form of $\hat{O}$ on three independent legs: the kinetic sectors are additive by the product geometry of the Hopf-fibred arena (commuting factor operators, summed spectrum, Proposition 2.1); the structure terms are additive by the Hamiltonian kinetic-plus-potential form, which is order-independent where an operad is not (Proposition 3.1); and the additive layer decomposition $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ is read off the ground state only under additive composition, a multiplicative composition yielding a product readout that is not α^{-1} (Theorem 4.1). **Verdict: PARTIAL** — additive composition is shown canonical; what is *not* delivered is an exhaustive exclusion of all operadic / non-additive compositions that might reproduce the same ground-state readout by another route. That exclusion is the named residual, recorded in Paper 18 §4.4 as **rmk:open:P018.5**. A294's re-typing of P18-T2 stands; sub-results (a) and (c) are Addenda 349b and 345b–348b; no canon .tex of P18 is edited beyond the open marker and no published number changes. Verifier: **verify_P350b.py**, 12/12 pass.

The three sub-results together. With (a) reduced to two named floors (349b), (b) shown canonical with an operadic-exclusion residual (here), and (c) reduced to one Schur scalar (345b–348b), the whole assembly-uniqueness question P18-T2 is now a short, explicit list of named floors rather than three open searches.