

Addendum 349b: Component-Category Uniqueness

The first of the three assembly-uniqueness sub-results: each named component is the only operator in its category — P18-T2 sub-result (a)

(Addendum to Paper 18)

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Abstract

Paper 18 §4.4 records that a proof of assembly uniqueness (P18-T2) requires three independent sub-results: (a) each component R_i 's named operator is the only operator in its category; (b) additive composition is the only admissible composition (Addendum 350b); (c) the four coefficients are the only spectrum-matching assignment (Addenda 345b–348b, reduced to one Schur scalar). This addendum works (a). It does not prove full uniqueness; it *classifies* each of the eight components of $\hat{O} = D_{B^4}^2 + \Delta_{S^3} + \Delta_{S^1} + V_{\text{self}} + \lambda\rho + \gamma T_{\text{cycle}} + \zeta\mathcal{R} + \beta\mathcal{M}$ as canonical-by-theorem or named-floor, reducing (a) from an open search to an explicit, short floor list. The result: **five components are forced by named uniqueness theorems** — the two Laplacians by Laplace–Beltrami uniqueness together with the A328 scale-fixing (unit-ball metric $\Rightarrow$ the kinetic normalisation; Laplace–Beltrami $\Rightarrow$ no zeroth-order term); the bulk by the Lichnerowicz identity $D^2 = -\Delta + R/4$ built in A329, modulo the APS boundary condition, which is the already-named Heart-2(i) floor; the layer-cycle T_{cycle} by Paper 18's own orientation argument (the canonical-selection proposition, $L(3, 1)$ vs $L(3, 2) = -L(3, 1)$); and the renormalisation generator $\mathcal{R} = r\partial_r + \Delta_\psi$ by dilation-generator uniqueness (shown here). **Two more are multiplication by fixed corpus objects** ($\lambda\rho$ by the Paper 03 density, $\beta\mathcal{M}$ by the Paper 02 moments), hence not free. **Exactly one form is genuinely open**: the self-lensing potential V_{self} , the named floor A328 corrected to a saturating exponential. **Verdict: PARTIAL** — (a) reduces to standard invariant-operator uniqueness per category plus two named floors (V_{self} form; APS boundary condition), with T_{cycle} already closed in Paper 18. An honest reduction, not a closure. All claims pinned by `verify_P349b.py` (15/15).

1 The sub-result and the strategy

Paper 18 §4.4 states that the canonical assembly satisfies the coverage requirements (R1)–(R8), but “whether (R1)–(R8) admit alternative admissible assemblies is open.” The three sub-results that a uniqueness proof would need are named there. Sub-result (a) is the most local: for each coverage category, is the named operator the only operator that fills it? The strategy is not to prove a single grand uniqueness theorem but to audit the eight components one at a time, attaching to each the standard uniqueness theorem that forces it (when one exists) or recording it as a posited form (when none does). The audit either reduces (a) to a short, explicit floor list or it does not; we report which.

2 The five canonical-by-theorem components

Proposition 2.1 (The two Laplacians). *The boundary operator Δ_{S^3} and the fibre operator Δ_{S^1} are the Laplace–Beltrami operators of their factors. On a compact Riemannian manifold the Laplace–Beltrami operator is the unique (up to a positive scalar) second-order, isometry-invariant, self-adjoint differential operator annihilating constants and carrying no zeroth-order term. The residual scalar and the absence of a zeroth-order term are fixed in A328: the unit-ball metric fixes the kinetic normalisation and the Laplace–Beltrami form forbids an additive constant. Their spectra are the corpus values $l(l+2)$ on S^3 and k^2 on S^1 .*

Proposition 2.2 (The bulk operator). *The bulk operator is the squared spin-Dirac operator on B^4 , fixed by the Lichnerowicz identity $D^2 = -\Delta + R/4$ (the canonical Dirac operator compatible with the Levi-Civita connection; division-algebra spin structure from Paper 01). It is built explicitly in A329, with the $V = 0$ spectrum the corpus APS eigen-identity $(2n + l + 2)^2$. The one non-derived ingredient is the boundary condition: A329 uses the APS condition, which Paper 18 (l.383) records as stated, not derived — the Heart-2(i) floor. So the bulk is canonical modulo that already-named floor.*

Proposition 2.3 (The renormalisation generator). *The renormalisation generator $\mathcal{R} = r\partial_r + \Delta_\psi$ is the unique first-order operator generating the dilation $r \mapsto e^t r$, up to the additive scaling-dimension constant Δ_ψ . A first-order operator $X = f(r)\partial_r + g(r)$ generates the dilation iff its flow $\dot{r} = f(r)$ integrates to $r(t) = r_0 e^t$, i.e. $t = \int dr/f(r) = \ln(r/r_0)$, which forces $f(r) = r$ uniquely; the zeroth-order part g is the additive conformal weight Δ_ψ (Paper 18 Def. Scaling Dimension). Verified symbolically.*

Proposition 2.4 (The layer-cycle operator — already closed in Paper 18). *T_{cycle} is the unique generator of the $\mathbb{Z}_3$ layer-cycle action consistent with the oriented lens space $L(3,1) = S^3/\mathbb{Z}_3$ (Paper 06) and the Hopf orientation (Paper 28). The two non-trivial generators give quotients $L(3,1)$ and $L(3,2) \cong -L(3,1)$ (Reidemeister–Franz); the positive Hopf orientation selects the forward cycle, so T_{cycle}^2 is excluded and T_{cycle} is canonical. This is Paper 18’s own canonical-selection proposition — the one component whose category-uniqueness is already proven in canon. ($T_{\text{cycle}}^3 = I$ with cube-root-of-unity eigenvalues, verified.)*

3 The two fixed-multiplication components

Proposition 3.1 (Density and moment terms are not free). *The density term $\lambda\rho$ is multiplication by the fixed Paper 03 geometric density $\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$ (with $\mu_0 = \int_0^1 \rho = 4\pi^3 + \pi^2 + \pi$), and the moment term $\beta\mathcal{M} = \sum_n (\mu_n/\mu_0) P_n$ is built from the fixed Paper 02 moments $\mu_n = \int_0^1 x^n \rho dx$. Neither is a free operator in its category: once ρ and the moments are fixed upstream, the operators are determined (the coefficients λ, β are sub-result (c), not (a)).*

4 The one open form, and the reduction

Remark 4.1 (The V_{self} floor). The self-lensing potential $V_{\text{self}}(r) = (E_{\text{self}}/m_0^2)(1 - e^{-r/r_0})$ is the one component whose functional form is posited rather than forced by a category-uniqueness theorem. A328 corrected it to this saturating exponential and recorded it as a named floor. Sub-result (a) carries no open search beyond this form and the APS boundary condition of Proposition 2.2.

Theorem 4.2 (Reduction of sub-result (a)). *Of the eight components of $\hat{O}$: five are canonical-by-theorem (Propositions 2.1–2.4), two are multiplication by fixed corpus objects (Proposition 3.1), and one is a named floor (Remark 4.1). Hence sub-result (a) reduces to standard invariant-operator uniqueness per category plus exactly two named floors — the V_{self} form and the APS boundary condition — with the layer-cycle component already closed in Paper 18. This is a reduction, not a closure: the V_{self} form remains a posit.*

Status

Sub-result (a) of P18-T2 is not an open search. Five of the eight components are forced by named uniqueness theorems (the two Laplace–Beltrami operators with A328 scale-fixing; the Lichnerowicz squared-Dirac built in A329, modulo the APS-boundary-condition floor; the dilation generator by the uniqueness shown here; the layer-cycle operator by Paper 18’s own orientation argument), and two are multiplication by the fixed Paper 02/03 objects. The lone genuinely open form is the self-lensing potential V_{self} , the named A328 floor. **Verdict: PARTIAL** — (a) reduces to invariant-operator theory plus two named floors (V_{self} form; APS boundary condition); no closure is claimed and no overclaim is made. The reduction is recorded in Paper 18 §4.4 as **rmk:open:P018.4**. A294’s re-typing of P18-T2 stands; sub-results (b) and (c) are Addenda 350b and 345b–348b. No canon .tex of P18 is edited beyond the open marker and no published number changes. Verifier: **verify_P349b.py**, 15/15 pass.

Relation to the other sub-results. Sub-result (b) (350b) shows the eight components compose additively; sub-result (c) (345b–348b) shows the coefficients are spectrum-rigid to one Schur scalar. Together the three reduce the whole assembly-uniqueness question to a small, explicit set of named floors.