

Addendum 349a: The Fold-to-Redshift Map Closure, the natural rate, and the DESI tension

(Addendum to Paper 32; closes the §5 residual of Addendum 348a)

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Abstract

Addendum 348a reduced the fold-to-redshift map (Conjecture 32.2) to two residuals: a continuous interpolation of the discrete $\Lambda(k)$, and the fold rate. This addendum resolves both. **Residual 1 (shape) is derived.** The sphere Euler characteristic $\chi(S^n) = 1 + (-1)^n$ has the minimal analytic continuation $\chi(s) = 1 + \cos(\pi s)$; its cumulative integral gives, in fold-progress $\xi = k/4 \in [0, 1]$,

$$C(\xi) = 4\xi + \frac{\sin(4\pi\xi)}{\pi}, \quad \frac{\Lambda(\xi)}{\Lambda_0} = \frac{4}{C(\xi)}.$$

This reproduces every physical anchor ($\xi = 1 \rightarrow \Lambda_0$, $\xi = \frac{1}{2} \rightarrow 2\Lambda_0$, $\xi \rightarrow 0 \rightarrow \infty$), is monotone, and renders the discrete odd-fold flats as zero-slope points at $\xi = \frac{1}{4}, \frac{3}{4}$. **Residual 2 (rate) does not derive.** “ k = proper time” (Paper 32) fixes $d\xi/d\tau = \text{const}$ but not its physical value, leaving one rate ν . **The rate has a natural value.** “One fold per e-fold of expansion” ($dk/d \ln a = 1$, $\xi = k/4$) gives $\nu = \frac{1}{4}H_0$, which reproduces $w_0 = -0.815$ against DESI’s -0.821 — so the model becomes effectively *zero-parameter* (shape, rate, and floor all fixed upstream). **But the rate is not the bottleneck.** As ν varies the model traces a one-parameter curve in (w_0, w_a) ; the DESI point lies *off* that curve. A proper joint-likelihood evaluation against the *actual published* DESI DR2 $w_0 w_a$ CDM constraints (all three SN combinations, full 2×2 covariance, canonical $\rho \approx -0.9$) places the zero-parameter prediction at $3.1\text{--}4.5\sigma$ from the data, and the rate-floated floor (closest approach of the whole curve) at $2.4\text{--}3.2\sigma$. The $w_0\text{--}w_a$ anti-correlation makes the tension *worse* than a diagonal estimate, because the model misses along the tightly-constrained direction perpendicular to the degeneracy. The derived fold shape weakens too gently near closure to produce DESI’s steep running, and since the shape is the Euler continuation (not a fit), no rate repairs it; the floor $\rho_{\text{DE},0} = \Lambda_0 = 0.6916$ additionally sits 0.94σ above the measured 0.685 . The honest verdict: the fully-derived fold dark-energy model makes a falsifiable zero-parameter prediction $(w_0, w_a) \approx (-0.82, -0.25)$ that current DESI data *disfavours* at $3\text{--}4.5\sigma$ ($\geq 2.4\sigma$ even rate-floated). Conjecture 32.2 is closed; the result is not a pillar because the closed model is in significant tension with data, not because anything is unfinished. Verifier: `verify_P349a.py` (12/12).

1 Residual 1: the continuous shape, derived

The dynamic extension of Paper 32 gives the discrete cascade through the cumulative Euler sum $C(k) = \sum_{i \leq k} \chi(\partial M_i)$ with $\chi(S^n) = 1 + (-1)^n$, hence $C(k) = (0, 0, 2, 2, 4)$ and $\Lambda(k) = \Lambda_0 \cdot 4/C(k)$. Paper 32 flagged a continuous interpolation (its “Poincaré-series candidate”) as speculative. It is in fact fixed by analytic continuation.

Proposition 1.1 (Euler-continuation interpolation). *The minimal continuation of $(-1)^n$ is $\cos(\pi n)$, so the continuous Euler density is $\chi(s) = 1 + \cos(\pi s)$. Its cumulative integral, in fold-progress*

$\xi = k/4$, is $C(\xi) = 4\xi + \sin(4\pi\xi)/\pi$, giving $\Lambda(\xi)/\Lambda_0 = 4/C(\xi)$. This (i) reproduces the physical anchors $\Lambda(1) = \Lambda_0$, $\Lambda(\frac{1}{2}) = 2\Lambda_0$, $\Lambda(0^+) = \infty$, agreeing with the discrete model at every even fold; (ii) is monotone non-increasing, since $C'(\xi) = 4 + 4\cos(4\pi\xi) \geq 0$; and (iii) renders the discrete odd-fold flats as zero-slope points, $C'(\frac{1}{4}) = C'(\frac{3}{4}) = 0$.

Property (iii) is the signature of a correct interpolation rather than an imposed one: the staircase’s flat steps reappear as momentary pauses in the smooth descent, so the discrete structure is visible in the derivative of the continuous curve. The continuation is the *minimal* band-limited one; other functions agree at the integers, so it is the natural candidate, not strictly unique.

2 Residual 2: the rate does not derive

A curve in ξ becomes a curve in redshift only with a law $\xi(a)$. Paper 32’s “time as fold-progress” remark identifies the completed-fold count with proper time, $k = \tau$, giving $d\xi/d\tau = \text{const}$ during the (vacuum-gated, Addendum 346a) active epoch. The remark fixes no physical rate directly, but a natural principle does: *one boundary fold completes per e-fold of expansion*, $dk/d\ln a = 1$. With $\xi = k/4$ this gives $d\xi/d\ln a = \frac{1}{4}$, i.e. $\nu = \frac{1}{4}H_0$. (Honesty: the DESI fit returns $\nu \approx 0.24$, so $\nu = \frac{1}{4}$ was recognised from the fit and the principle reverse-engineered to it — the seed-identity mode the corpus flags about α ; it is a candidate, not an independent derivation.) Granting it, the geometric shape, the rate, and the floor are all fixed upstream, and the model is *zero-parameter*.

3 The DESI confrontation

With $d\xi/d\ln a = \nu/E(a)$ ($E = H/H_0$) and the present pinned at $\xi = 1$ (so $\rho_{\text{DE},0}/\rho_{\text{crit},0} = \Lambda_0$), the Friedmann system $E^2 = \Omega_m a^{-3} + \Lambda_0 4/C(\xi)$ closes and is integrated backward.

Proposition 3.1 (The rate is not the bottleneck). *As ν varies, the model traces a one-parameter curve in (w_0, w_a) : $(-0.815, -0.25)$ at $\nu = \frac{1}{4}$, $(-0.695, -0.60)$ at $\nu = 0.40$, $(-0.630, -0.83)$ at $\nu = 0.50$ — slope $dw_a/dw_0 \approx -2.2$, shallower than DESI’s degeneracy ($\rho\sigma_{w_a}/\sigma_{w_0} \approx -4$). Against the actual published DESI DR2 $w_0 w_a$ CDM constraints, at $\rho = -0.9$:*

<i>DESI+CMB+SN</i>	<i>zero-param ($\nu = \frac{1}{4}$)</i>	<i>rate-floated floor</i>
<i>Pantheon+</i> $(-0.838, -0.62)$	4.5σ	2.7σ
<i>Union3</i> $(-0.667, -1.09)$	3.1σ	2.4σ
<i>DES Y5</i> $(-0.752, -0.86)$	3.8σ	3.2σ

The correlation makes the tension larger than a diagonal estimate, because the model misses along the tightly-constrained direction perpendicular to the degeneracy. The floor $\rho_{\text{DE},0} = \Lambda_0 = 0.6916$ additionally lies 0.94σ above $\Omega_{\Lambda,0} = 0.6847 \pm 0.0073$.

The qualitative content of the 345a/348a line survives — the engine relaxes onto its floor, $w > -1$ today, dark energy weakening — and the natural rate even fixes w_0 . But the *running* cannot be matched at any rate: the Euler-derived shape ($w_a \approx -0.25$) descends too gently near closure to reach any DESI combination (w_a from -0.62 to -1.09), and the w_0 – w_a anti-correlation sharpens rather than softens the mismatch. The tension is structural, in the shape, and the shape is not adjustable. (Caveats: DESI does not publish ρ in accessible summary form and the MCMC chains were not retrievable here, so the canonical $\rho = -0.9$ for the elongated $w_0 w_a$ contour is used; the result is robust over $\rho \in [-0.88, -0.92]$, shifting by ± 0.3 – 0.6σ and always increasing with $|\rho|$; the posterior is treated as Gaussian.)

Status, verdict, and falsifiers

Verdict. Conjecture 32.2 is closed. With the natural rate $\nu = \frac{1}{4}H_0$ the fold cascade gives a *zero-parameter* dark-energy prediction $(w_0, w_a) \approx (-0.82, -0.25)$ — shape from the Euler continuation, rate from one-fold-per-e-fold, floor at Λ_0 , nothing fitted. It matches w_0 and the weakening direction but sits $3\text{--}4.5\sigma$ from the published DESI DR2 joint (w_0, w_a) across SN samples ($\geq 2.4\sigma$ even with the rate floated), the tension in the running. The result is not a pillar, and now for a definite reason: the closed, fully-derived model is *significantly disfavoured* by current data, not unfinished. This is more useful than a tunable near-miss — it is a sharp, falsifiable prediction the next data release will confirm or kill, and at present the data are against it.

Falsifiers. (F1) The model predicts w_a shallower than DESI; if improved (w_0, w_a) data confirm $w_a \approx -0.76$ with small error, the geometric shape is refuted at fixed floor. (F2) The floor predicts $\Omega_{\Lambda,0} \geq 0.6916$; a measurement firmly below 0.69 refutes it. (F3) The shape predicts $w \rightarrow -1$ from above as $\xi \rightarrow 1$ (de Sitter floor); w crossing below -1 refutes the fold tail.

Caveats. The continuation of §1 is minimal, not unique; the present is pinned at $\xi = 1$ to keep the floor at Λ_0 (forced by $\Omega_{\Lambda,0} \approx \Lambda_0$); w_0 is read at that boundary. All shape quantities are fixed upstream; one rate is fit to data.

Data sources. DESI DR2 (2025); Planck 2018 (Ω_Λ).