

Addendum 348a: Cosmology from the Fold Cascade

Inflation, dark energy, and the DESI/Planck confrontation

(Addendum to Paper 32; consolidates the 345a–347a line)

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Abstract

This addendum consolidates one cosmological line of the corpus. From the fold-progress dark-energy cascade of Paper 32 it draws a single synthesis and runs two confrontations. **Synthesis:** the cascade $B^4 \rightarrow S^3 \rightarrow S^2 \rightarrow S^1 \rightarrow S^0$, whose dark-energy density runs from $\Lambda = \infty$ at the pre-fold stage to $\Lambda_0 = 1 - \pi^2/32$ at completion, is a single declining engine — inflation its violent early end, dark energy its exhausted tail — unifying three prior conjectures (Papers 20, 27, 5). **Dark-energy confrontation:** against DESI DR2 the robust signal (dark energy weakening now) is the engine relaxing onto its floor; the integer fold step ($\times 2$) overshoots the observed $+7\%$ swing by $\sim 14\times$, forcing “now” to $\sim 93\%$ through the final fold (Theorem 2.1); a golden self-similar rate is refuted (the breath rotation number is near-rational, not noble); and the fraction-versus-density unit question is closed (Proposition 3.1). **Inflation confrontation:** the fold-to-redshift map does not close — it reduces to one posit and is reframed as vacuum-gated, with the matter era a dormant gap (§4) — but the *canonical* self-lensing potential of Paper 18, $V_{\text{self}} = (E_{\text{self}}/m_0^2)(1 - e^{-r/r_0})$, a saturating plateau, yields $n_s = 1 - 2/N = 0.9649$ at $N = 57$, an exact match to Planck, with $r \sim 10^{-8}$ (§5). No quantities are fit. The dynamic- Λ spine remains Conjecture 32.2; this is a dark-sector continuation, not a new pillar. Verifier: `verify_P348a.py` (26/26).

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1 The fold cascade as one declining engine

Three corpus statements describe one object from different ends. Paper 20 §7.2 conjectures that “the breathing dynamics $C \circ P = I$ at early times drives inflation.” Paper 27 §6 reads the cosmic arc as the descent from Ω , with dark energy “the overhead cost of maintaining consensus across

expanding space.” Paper 5 lists “inflation from geometric oscillation dynamics.” Paper 32’s fold-progress table closes the loop quantitatively: with $\chi(S^n) = 1 + (-1)^n$ and the cumulative Euler sum $C(k) = \sum_{i \leq k} \chi(\partial M_i)$,

$$\Lambda(k) = \Lambda_0 \cdot \frac{4}{C(k)} : \quad \infty, \infty, 2\Lambda_0, 2\Lambda_0, \Lambda_0 \quad (k = 0, 1, 2, 3, 4), \quad C(k) = (0, 0, 2, 2, 4).$$

Proposition 1.1 (Cascade unification). *Inflation and dark energy are the two ends of the single fold cascade: the early, divergent stages ($\Lambda \gg \Lambda_0$) are the inflationary epoch, and the final stage’s relaxation onto Λ_0 is the present dark-energy era. The proposition names one engine where the corpus carried three separate conjectures; it is qualitative and inherits their conjecture status.*

The static value is itself a zero-parameter geometric result: $\Lambda_0 = 1 - \pi^2/32 = 0.6916$ is the fraction of the bounding 4-cube not covered by the unit 4-ball (Paper 32; the Cantor-bridge addendum shows the cube is the pre-geometric container of $\mathbb{R}^4$, so the choice is forced). It matches Planck’s $\Omega_\Lambda = 0.6847 \pm 0.0073$ at 0.94σ .

2 The dark-energy confrontation

DESI DR2 (CMB+DESI+SN, $w_0 w_a$ CDM) reports $w_0 = -0.821 \pm 0.065$, $w_a = -0.76_{-0.26}^{+0.31}$, time variation preferred at $2.8\text{--}4.2\sigma$, with $w > -1$ today and $w < -1$ in the recent past. The density history for $w(a) = w_0 + w_a(1 - a)$ is $\rho_{\text{DE}}(a)/\rho_{\text{DE},0} = a^{-3(1+w_0+w_a)} \exp[-3w_a(1 - a)]$.

Direction. This is modestly higher in the recent past and declining toward today — the engine relaxing onto its floor. The apparent deep-past rise is the linear- $w(a)$ extrapolation beyond the DESI range ($z \lesssim 2$) and is the least robust part; the robust statement is the recent decline, which is fold-faithful.

Magnitude and near-closure. The integer cascade changes Λ by $\times 2$ across an active fold (+100%); the observed density bump peaks at only +7.1% near $z \approx 0.31$. The fold step overshoots by $\sim 14\times$.

Theorem 2.1 (Near-closure bound). *If the present evolution is the tail of the final fold, the visible residual amplitude ($\sim 7\%$) as a fraction of a full fold step (100%) places “now” at $1 - 0.071/1.00 \approx 0.93$, i.e. $\sim 93\%$ through the final fold. The factor-of-two step is reconciled with the small observed evolution only under near-closure; this is a consequence of the magnitude mismatch, not an assumption.*

Golden-ratio refutation. The cascade is nesting spheres, and the golden ratio is the fixed point of self-similar nesting, so a golden fold rate is the natural conjecture. It is excluded by the corpus’s own clock: the breath rotation number $\rho = (\pi\alpha^{-1}) \bmod 1 = 0.5122$ has continued fraction $[0; 1, 1, 19, 1, 10, \dots]$ with large partial quotients — a near-rational beside the tritone $\frac{1}{2}$ — whereas $\varphi = [0; 1, 1, 1, \dots]$ is the maximally irrational noble number. The native self-similar ratio is 2 (halving); φ lives in the E8 sector (Addenda 94/95), not the cascade. A golden step (+62%) still overshoots the observed +7% by $\sim 9\times$.

Location of “now”. At full closure the fold function f ($\Lambda = \Lambda_0 f$, $f(1) = 1$, $f > 1$ for $x < 1$) is at its minimum, so $f'(1) = 0$ and $w = -1$: no evolution. DESI’s $w_0 = -0.821 \neq -1$ at $\gtrsim 3\sigma$ forces $x_{\text{now}} < 1$. “Now = closure” is a definitional placement, contradicted by the data; Λ_0 is the future asymptote, not the present value.

3 The fraction–density reconciliation

The static $\Lambda_0 \approx 0.69$ is a volume *fraction* (≤ 1), yet the dynamic $\Lambda(2) = 2\Lambda_0 \approx 1.383$ exceeds 1. They are different quantities.

Proposition 3.1 (Reconciliation). *With $\rho_{\text{crit},0} = 3H_0^2/8\pi G$: (i) the static residual is the present fraction, $\Lambda_0 = \Omega_{\Lambda,0} = \rho_{\Lambda,0}/\rho_{\text{crit},0} \leq 1$; (ii) the fold factor normalised to the present is the density evolution, $\rho_{\Lambda}(k)/\rho_{\Lambda,0} = (4/C(k))/(4/C(4)) = 4/C(k)$; (iii) hence $\Lambda(k) = \Lambda_0 \cdot 4/C(k) = \rho_{\Lambda}(k)/\rho_{\text{crit},0}$, a density in present-critical units, which may exceed 1 in the past without being a fraction. The boundary condition $\Lambda(4) = \Lambda_0$ is the identity “present density = present fraction.”*

For any matter density the epoch fraction $\Omega_{\Lambda} = \rho_{\Lambda}/(\rho_{\Lambda} + \rho_m + \dots) \leq 1$ automatically; the half-fold density 1.383 gives $\Omega_{\Lambda} = 0.815$ at the $z = 0$ floor and less earlier. The confrontation of §2 was density-to-density throughout, so it is well-posed.

4 The fold-to-redshift map: reduction and vacuum gating

A quantitative $\Lambda(z)$ curve requires a fold coordinate and a law to a . The corpus offers three fold coordinates, never identified: **(A)** the boundary-collapse index k ; **(B)** the self-lensing x in Paper 20’s $H^2 \sim e^{-x/\kappa}$; **(C)** the Cantor-bridge growth phase $\Theta(x) = 4\pi^3 x^4 + \pi^2 x^3 + \pi x^2$, $\Theta(1) = \alpha^{-1}$.

Proposition 4.1 (Non-closure and vacuum gating). *(i) The map does not close from committed structure: (A), (B), (C) are unbridged and no global law to a is supplied; closing it requires an identification $A=B=C$ and a dynamical law, both new objects. (ii) The one dynamical link, $H^2 \sim e^{-x/\kappa}$, is vacuum-specific: in matter domination $H^2 \propto a^{-3}$ tracks matter, not the fold. Hence the fold-clock advances only when vacuum-dominated — inflation and the present dark-energy era — and is dormant through the matter era. The map is piecewise; the matter era is a gap, not a fold.*

This reduces Conjecture 32.2 to (i) a canonical identification $A=B=C$ and (ii) one posit (that the inflation law governs both vacuum epochs). Under it, near-closure sharpens to $\Delta x = \kappa \ln 2 \approx 1.5 \times 10^{-3}$ of fold-progress.

5 The inflation sector from the canonical potential

Paper 18 §3 fixes the self-lensing potential of the master operator as

$$V_{\text{self}}^{\text{can}}(r) = \frac{E_{\text{self}}}{m_0^2} (1 - e^{-r/r_0}), \quad r_0 = \kappa = \alpha^{5/4} = 0.00213, \quad E_{\text{self}} = 13.177. \quad (1)$$

This is a *saturating plateau* (concave, $V'' < 0$), the observationally favoured class — not the convex decaying heuristic. Taking the inflaton to be the canonically normalised radial field and $u = e^{-\phi/r_0} \ll 1$ on the plateau: $\epsilon \approx u^2/2r_0^2$, $\eta \approx -u/r_0^2$, and $N \approx r_0^2/u_*$, so $u_* = r_0^2/N$ and $\eta_* = -1/N$.

Theorem 5.1 (Plateau-attractor spectral index). *For (1), $n_s = 1 + 2\eta_* - 6\epsilon_* = 1 - 2/N$ to leading order on the plateau, independent of r_0 ; the tensor-to-scalar ratio is $r = 16\epsilon_* = 8r_0^2/N^2$.*

Corollary 5.2 (Planck match). *$n_s = 1 - 2/N = 0.9649$ at $N = 57$ e -folds (standard window), an exact match to Planck 0.9649 ± 0.0042 ; across $N \in [50, 60]$, $n_s \in [0.960, 0.967]$ (all $\lesssim 1.2\sigma$). With $r_0 = \kappa$, $r = 8r_0^2/N^2 \approx 1.1 \times 10^{-8}$ — effectively no primordial gravitational waves.*

This is the universal value of Starobinsky, Higgs, and α -attractor inflation: the plateau shape, not the detailed r_0 , fixes n_s . The naive decaying heuristic gives $n_s \approx 0.99$ ($\sim 5\sigma$); using the correct potential removes the tension entirely.

6 Status, falsifiers, and what would earn Paper status

Standing. Two falsifiable confrontations now attach to the cascade: the dark-energy near-closure (Theorem 2.1) and the inflation index $n_s = 1 - 2/N$ (Corollary 5.2). The fraction–density floor is closed (Proposition 3.1). No quantities are fit: Λ_0 , the fold step, ρ , κ , E_{self} , and the form (1) are fixed upstream; DESI and Planck enter only as targets.

Open. The synthesis (Proposition 1.1) is conjecture-tier; the fold-to-redshift map is reduced, not closed (Proposition 4.1); the dynamic- Λ spine remains Conjecture 32.2. This work is a continuation of the dark-sector line (Papers 32/38/39), not a new pillar.

Falsifiers. (F1) Dark energy must keep weakening toward $\Lambda_0 \approx 0.6916$ and halt ($w \rightarrow -1$ from above); w crossing below or oscillating refutes the fold tail. (F2) n_s outside $[0.960, 0.967]$ refutes the plateau prediction. (F3) A tensor detection at $r \gtrsim 10^{-3}$ refutes the $r \sim 10^{-8}$ prediction.

Path to Paper status. A canonical identification $A=B=C$ would close Conjecture 32.2, turning the fold-clock from a posited into a *derived* cosmological coordinate, with the two confrontations following from first principles. That — new derived structure, not synthesis over an open conjecture — is what would re-cut this material as a standalone Paper.

Data sources. DESI DR2 (2025); Planck 2018 (Ω_Λ , n_s , r with BICEP/Keck).