

Addendum 347a: The Inflation Sector from the Canonical Self-Lensing Potential

$n_s = 1 - 2/N$ and the resolution of the Addendum 346a tension
(Addendum to Papers 18, 20, 32)

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Abstract

Addendum 346a reduced the fold-to-redshift map to a single dynamical posit and tested its inflation-sector consequence using the heuristic decaying exponential $H^2 \sim e^{-x/\kappa}$ of Paper 20 §7.2, finding a 5.3σ tension in the scalar spectral index ($n_s = 0.987$ against Planck’s 0.9649). That tension was an artefact of the heuristic. The *canonical* self-lensing potential is given in Paper 18 §3 as

$$V_{\text{self}}^{\text{can}}(r) = \frac{E_{\text{self}}}{m_0^2} (1 - e^{-r/r_0}), \quad r_0 = \kappa = \alpha^{5/4},$$

a *saturating* potential whose shape is concave (a plateau), not the convex decay used in Addendum 346a. Plateau potentials are the observationally favoured class. We compute its slow-roll observables and find the universal plateau-attractor result $n_s = 1 - 2/N$, giving $n_s = 0.9649$ at $N = 57$ e-folds — an exact match to Planck across the standard window $N \in [50, 60]$ ($n_s \in [0.960, 0.967]$) — with a tensor-to-scalar ratio $r = 8r_0^2/N^2 \approx 10^{-8}$, far below any foreseeable detection. The Addendum 346a inflation tension is removed by using the correct potential; the model’s standing in the inflation sector upgrades from strained to a clean, falsifiable prediction. Verifier: `verify_P347a.py` (11/11).

1 The correct potential

Paper 18 §3 fixes the self-lensing potential of the master operator as

$$V_{\text{self}}^{\text{can}}(r) = \frac{E_{\text{self}}}{m_0^2} (1 - e^{-r/r_0}), \quad r_0 = \kappa = \alpha^{5/4} = 0.00213, \quad E_{\text{self}} = 13.177. \quad (1)$$

This rises from 0 at $r = 0$ to the plateau E_{self}/m_0^2 as $r \rightarrow \infty$: it is *concave* ($V'' < 0$). The heuristic $H^2 \sim e^{-x/\kappa}$ used in Addendum 346a is the opposite shape — a *convex*, monotonically decaying exponential — and convex/decaying potentials generically overproduce n_s (or r). The substitution of (1) for the heuristic is the whole content of this addendum.

2 Slow-roll on the plateau

Take the inflaton to be the canonically normalised radial field $\phi \equiv r$ ($M_{\text{Pl}} = 1$) and write $u = e^{-\phi/r_0}$. From (1),

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2 = \frac{u^2}{2r_0^2(1-u)^2}, \quad \eta = \frac{V''}{V} = -\frac{u}{r_0^2(1-u)}.$$

On the plateau ($u \ll 1$): $\epsilon \approx u^2/2r_0^2$ and $\eta \approx -u/r_0^2$, so $|\eta| \gg \epsilon$. The e-fold count is

$$N = \int \frac{V}{V'} d\phi \approx \frac{r_0^2}{u_*}, \quad \text{hence} \quad u_* = \frac{r_0^2}{N}, \quad \eta_* = -\frac{1}{N}.$$

Theorem 2.1 (Plateau-attractor spectral index). *For the canonical self-lensing potential (1),*

$$n_s = 1 + 2\eta_* - 6\epsilon_* = 1 - \frac{2}{N}$$

to leading order on the plateau, independent of r_0 . The tensor-to-scalar ratio is $r = 16\epsilon_ = 8r_0^2/N^2$, which does depend on r_0 .*

The n_s result is the same universal value carried by Starobinsky inflation, Higgs inflation, and the α -attractors: the plateau shape, not the detailed r_0 , fixes it.

3 Planck confrontation

Proposition 3.1 (Match to Planck). *$n_s = 1 - 2/N$ equals the Planck 2018 central value 0.9649 ± 0.0042 at $N = 57$ e-folds, within the standard reheating window. Across $N \in [50, 60]$, $n_s \in [0.960, 0.967]$, all within $\sim 1.2\sigma$. The tensor-to-scalar ratio is*

$$r = \frac{8r_0^2}{N^2} \approx 1.1 \times 10^{-8} \quad (N = 57, r_0 = \kappa),$$

far below the bound $r < 0.06$ and below any foreseeable sensitivity: the model predicts effectively no primordial gravitational waves.

Corollary 3.2 (Resolution of Addendum 346a, F1). *The 5.3σ tension reported in Addendum 346a used the convex heuristic $e^{-x/\kappa}$, which gives $n_s \approx 0.99$. Replacing it with the canonical concave potential (1) gives $n_s = 1 - 2/N = 0.9649$. The tension was a property of the heuristic, not of the corpus; Addendum 346a's falsifier F1 is discharged.*

Status, falsifiers, and caveats

Status. The inflation sector of the fold cascade now carries a clean, parameter-free spectral prediction $n_s = 1 - 2/N$ and a definite (tiny) tensor amplitude. No quantities are fit: $r_0 = \kappa = \alpha^{5/4}$, $E_{\text{self}} = 13.177$, and the form (1) are fixed upstream in Paper 18; Planck data enter only as the target.

Falsifiers. (F1') A future measurement pinning n_s outside $[0.960, 0.967]$ (the $N \in [50, 60]$ band) falsifies the plateau prediction. (F2') A detection of primordial tensors at $r \gtrsim 10^{-3}$ falsifies the $r = 8r_0^2/N^2$ prediction (with $r_0 = \kappa$): this model lives at $r \sim 10^{-8}$.

Caveats. (i) The computation takes the inflaton to be the canonically normalised B^4 radial field; the $n_s = 1 - 2/N$ result is robust to the value of r_0 , but a non-flat field-space metric would rescale the map between ϕ and r and could shift r (not n_s). (ii) $n_s = 1 - 2/N$ is the prediction; pinning the central 0.9649 requires $N \approx 57$, which depends on the reheating history and is not independently derived here. (iii) The estimate is leading-order slow-roll; higher-order corrections are below the Planck error bar at these ϵ, η .

Data sources. Planck 2018 ($n_s = 0.9649 \pm 0.0042$; $r < 0.06$ with BICEP/Keck 2018).