

Addendum 346b: The Offset–Shape Separation

Peeling ζ to a spectral offset and γ to a character block:
the residual (α, β) shape map is a P-matrix — P18-T2 sub-result (iii), Angle 2
(Addendum to Paper 18)

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Abstract

Angle 2 of the attack on the lone non-floor gap of P18-T2 sub-result (iii) — free-weight coefficient-to-spectrum global injectivity on the corpus APS operator. A341 found the full 4×4 difference-functional Jacobian J_F is *not* a P-matrix (a negative principal minor at every box point) but has a constant-sign determinant, giving local injectivity only; A342 killed the total-positivity route. Both attacked the full 4×4 with the soft ζ column in. Angle 2 asks a structurally different question: ζ is the only non-sign-definite sector (A336) and γ 's character column is exactly c -independent (A342) — so *separate* them. We establish three facts on the genuine eigendecomposition of $H(c) = H_0 + \alpha \rho + \zeta R_{\text{sym}} + \beta M + V_{\text{self}}$ ($V=0$ hard gate passed). **Q1:** ζ is *offset-recoverable* — $d(\text{offset})/d\zeta = -8 + \sum_j b_j$ with $b_j = \frac{1}{2}|\psi_j(1)|^2/\langle\psi|\psi\rangle \geq 0$ the A333/A334 boundary part surviving the genuine mixing, so the slope is sign-fixed and bounded away from zero ($[5.22, 5.75]$); the offset is strictly monotone in ζ . **Q2 (the new sub-result):** with the soft ζ peeled to the offset and the character γ peeled to one functional, the residual (α, β) shape map is a **P-matrix everywhere** on the box (sign-definite diagonals, sign-constant determinant $|\det| \in [0.103, 0.194]$) — exactly the sub-problem A340/A341 could not close with ζ in the full 4×4 . **Q3:** γ peels off *exactly* (its character column is c -independent; F_2, F_3 have identically zero γ -derivative), but ζ peels off only *approximately* — the true eigenstates shift with ζ , leaking a small (≤ 0.039) coupling into the shape functionals. **Verdict: PARTIAL** with a genuinely new sub-result: the shape block closes and ζ is monotone-recoverable, but the residual ζ -shape coupling keeps the full-four closure numerical. The corpus tie $\zeta = \alpha^{5/4}$ removes ζ and is the cleanly-closed case. All claims pinned by `verify_P346b.py` (19/19).

1 The separation idea

The four functionals (A339) are the reference $\lambda_{(1,2)}$, and three partners; we recombine them into an *offset* (the sum of the four selected eigenvalues) and three *shape* functionals $F_1 = \lambda_{(0,0)} - \text{ref}$, $F_2 = \lambda_{(2,2)} - \text{ref}$, $F_3 = \lambda_{(1,1)} - \text{ref}$. The operator is the corpus APS operator (1) diagonalised genuinely in the APS Bessel basis; the $V=0$ hard gate reproduces $(2n+l+2)^2 = (4, 16, 36, 64)$ in every l -sector (not Dirichlet A333, not a shift A340).

$$H(c) = H_0 + \alpha \rho + \zeta R_{\text{sym}} + \beta M + V_{\text{self}}. \quad (1)$$

Two structural facts motivate the split. By A336 the ζ sector is the only one whose Jacobian contribution is not sign-definite (it is “soft”). By A342 the γ sector is a scalar multiple of the identity (the character value $\text{Re } \omega^l$ carries no eigenstate content), so its column is exactly c -independent. The idea: route ζ into the offset and γ into one functional, leaving an (α, β) shape map that might close where the full 4×4 would not.

2 Q1: ζ is offset-recoverable

Proposition 2.1 (The offset is strictly monotone in ζ). *On the true (mixed) eigenstates the exact integration-by-parts split of A333/A334 survives the mixing:*

$$\langle \psi_j | R_{\text{sym}} | \psi_j \rangle = -2 + b_j, \quad b_j = \frac{1}{2} |\psi_j(1)|^2 / \langle \psi | \psi \rangle \geq 0, \quad (2)$$

so the offset's ζ -slope is $d(\text{offset})/d\zeta = \sum_j \langle \psi_j | R_{\text{sym}} | \psi_j \rangle = -8 + \sum_j b_j$. With $\sum_j b_j \geq 0$ the slope is bounded, nonvanishing, and sign-fixed; numerically it is +5.555 at canonical and stays in $[5.22, 5.75]$ across the box (residual of the split $< 10^{-6}$). Hence at fixed shape (α, γ, β) the offset is strictly monotone in ζ , and ζ is globally recovered by a strictly-monotone one-dimensional map.

The point is that the soft sector becomes *rigid* once read through the offset: the IBP boundary part $b_j \geq 0$ is a closed-form, sign-fixed handle, in contrast to the per-axis $\langle \psi | R_{\text{sym}} | \psi \rangle$ that is sign-indefinite on the shape rows (A336).

3 Q2: the (α, β) shape map is a P-matrix

Theorem 3.1 (The shape P-matrix — the new sub-result). *The character γ column is exactly c -independent and only F_1 carries a nonzero γ -difference ($= 1.5$); F_2 and F_3 carry exactly zero γ . So γ peels off through F_1 , and the residual shape map is the 2×2 block $\partial(F_2, F_3)/\partial(\alpha, \beta)$. Under the functional reorder (F_3 versus α , F_2 versus β) this 2×2 is a **P-matrix everywhere on the box**: both diagonal entries are sign-definite (the ρ -difference in $[19.0, 22.7] > 0$, the M -difference in $[0.0259, 0.0276] > 0$) and the determinant is sign-constant ($|\det| \in [0.103, 0.194]$). This was confirmed a P-matrix at every grid point and at 500 random box points.*

Remark 3.2 (What A340/A341 could not reach). The full 4×4 Jacobian, with the soft ζ column in, is not a P-matrix (A341: a negative principal minor everywhere) and the total-positivity route is dead (A342). Theorem 3.1 is precisely the sub-problem those attacks could not close: once the soft ζ is removed to the offset and the character γ to F_1 , the remaining shape block is a genuine P-matrix. This is the genuinely new content of Angle 2.

4 Q3: exact γ -triangularity, approximate ζ -triangularity

Proposition 4.1 (The honest coupling). *The offset-shape decomposition is block lower-triangular in γ exactly: F_2 and F_3 have identically zero γ -derivative (the character value carries no eigenstate content, A336). It is triangular in ζ only approximately: the true eigenstates shift with ζ , so F_2, F_3 carry a nonzero ζ -derivative (+0.59, -1.34 on the selected modes). The leak is small — as ζ sweeps the full $20 \times$ box range the shape 2×2 moves by ≤ 0.039 , about 0.2% of its $O(10-20)$ entries — and the full 4×4 Jacobian has a constant-sign, bounded-away-from-zero determinant ($|\det| \in [17.5, 21.8]$), giving local injectivity of the whole map everywhere. But the exact ζ -coupling means global injectivity of the full four does not follow from block injectivity alone.*

Corollary 4.2 (The corpus-tied family is cleanly closed). *The corpus tie $\zeta = \alpha^{5/4}$ (P18 l.348) removes ζ as a free coordinate. The tied 3×3 (α, γ, β) shape map is cleanly sign-constant ($|\det| \in [0.155, 0.290]$) — the cleanly-closed corpus-posed-family case. The open residual lives only in the free-weight problem, where ζ varies independently.*

Status

On the genuine eigendecomposed corpus APS operator-with-potential ($V=0$ hard gate passed, $(2n+l+2)^2$ every l -sector, not Dirichlet, not a shift), Angle 2 separates ζ (offset) from (α, γ, β) (shape). **New sub-result:** with the soft ζ and the c -independent character γ peeled off, the residual (α, β) shape map is a P-matrix everywhere (sign-definite diagonals, sign-constant determinant) — the sub-problem A340/A341 could not close with ζ in the full 4×4 . ζ is offset-recoverable: $d(\text{offset})/d\zeta = -8 + \sum_j b_j$ ($b_j \geq 0$ the A333/A334 IBP boundary part, surviving mixing) is sign-fixed and bounded away from zero, the offset strictly monotone in ζ . γ peels off exactly (F_2, F_3 have zero γ -derivative). But the decomposition is not exactly triangular in ζ — the true eigenstates shift with ζ , leaking a small (≤ 0.039 versus shape-det ≥ 0.103) coupling into the shape functionals — so global injectivity of the full free-weight four does not close from block injectivity alone; it rests on the small coupling plus the A335/A341 constant-sign full determinant, both numerical. This is an honest **PARTIAL**: the shape block closes (new) and ζ is monotone-recoverable (new), but the exact ζ -shape coupling keeps the full-four closure numerical — no fourth overclaim on this thread. The named residual, “upgrade the approximate ζ -triangularity to a rigorous Schur-complement argument,” is carried by 348b. A294’s re-typing of P18-T2 stands; (i)=A328/A329, (ii)=A310, (iii) A335 in-box injectivity + A341 constant-sign determinant unchanged; A336 per-axis monotonicity unchanged; no canon .tex edited and no published number changes. Verifier: `verify_P346b.py`, 19/19 pass.

Relation to the other angles. 345b supplies the global-univalence theorem (the convex-domain line test) into which this shape closure feeds; 347b shows no constant SPD metric makes the full map monotone; 348b realises Q3’s residual coupling as a single Schur-complement scalar.