

Addendum 346a: The Fold-to-Redshift Map

Vacuum-regime gating and the reduction of Conjecture 32.2

(Addendum to Paper 32; sequel to Addendum 345a)

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Abstract

Addendum 345a left one floor load-bearing: the fold-to-redshift map (Conjecture 32.2), whose closure would turn the fold-progress dark-energy cascade from a direction-plus-bound into a quantitative $\Lambda(z)$ curve. This addendum does *not* close it. It does three things that sharpen it. First, it locates the obstruction: the corpus carries *three* distinct, unbridged fold-parametrizations — the boundary-collapse index k (Paper 32 dynamic extension), the self-lensing parameter x of the inflation law $H^2 \sim E_{\text{self}} e^{-x/\kappa}$ (Paper 20 §7.2), and the radial growth phase $\Theta(x)$ of the Cantor bridge (Paper 32 Addendum, $\Theta(1) = \alpha^{-1}$) — and Paper 20 states outright that no bridge between them is supplied. Second, it reframes the conjecture: the one dynamical link on offer is *vacuum-regime-specific*, since in matter domination $H^2 \propto a^{-3}$ tracks matter rather than the fold; therefore the fold-clock couples to expansion only when vacuum-dominated, and the map is piecewise — two active branches (inflation, present dark energy) bracketing a *dormant matter-era gap*. The matter era is a gap, not a fold. Third, it reduces the open problem to a single named posit P (that the inflation law governs both vacuum epochs) and tests P 's independent CMB consequence: with $\kappa = \alpha^{5/4} = 0.00213$ and $\epsilon \sim \kappa$, the tensor-to-scalar ratio $r = 16\epsilon = 0.034$ sits under the observational bound, while $n_s = 1 - 6\epsilon = 0.987$ is 5.3σ high against Planck — a real tension, recorded not buried. Under P , near-closure sharpens to $\Delta x = \kappa \ln 2 \approx 1.5 \times 10^{-3}$ of fold-progress. Verifier: `verify_P346a.py` (12/12).

1 The obstruction: three unbridged parametrizations

A fold-to-redshift map requires a fold coordinate and a law relating it to the scale factor a . The corpus offers three fold coordinates, never identified with one another.

- (A) **Boundary-collapse index k .** The dynamic extension of Paper 32 indexes the cascade $B^4 \rightarrow S^3 \rightarrow S^2 \rightarrow S^1 \rightarrow S^0$ by completed folds k , with $\Lambda(k) = \Lambda_0 \cdot 4/C(k)$ and $C(k) = (0, 0, 2, 2, 4)$. Discrete.
- (B) **Self-lensing parameter x .** Paper 20 §7.2 conjectures that breathing dynamics drive inflation through $H^2 \sim E_{\text{self}} e^{-x/\kappa}$, $\kappa = \alpha^{5/4}$. Continuous, but “a conjectural cosmological use of V_{self} ; no bridge to P18's $V_{\text{self}}^{\text{can}}$ branch is supplied.”
- (C) **Radial growth phase $\Theta(x)$.** The Cantor-bridge addendum carries the accumulated phase $\Theta(x) = 4\pi^3 x^4 + \pi^2 x^3 + \pi x^2$ of the ball growing inside the cube, with $\Theta(1) = \alpha^{-1}$. Continuous.

Proposition 1.1 (Non-closure from committed structure). *The map cannot be closed from committed corpus structure alone. Coordinates (A), (B), (C) are not identified with one another in any corpus result, and no global law relating any of them to a is supplied. Closing the map requires both*

an identification ($A=B=C$, or a canonical reduction among them) and a dynamical law; each is a new object. Per the criterion stated in the Paper 32 dynamic addendum — “whether the mapping closes without introducing new objects determines whether this is an addendum or a new paper” — the map does not close without new structure.

2 The reframing: vacuum-regime gating

The single dynamical link the corpus offers, (B)’s $H^2 \sim e^{-x/\kappa}$, is not a candidate for a *global* $x(a)$. It is vacuum-regime-specific.

Proposition 2.1 (Vacuum gating). *During matter (or radiation) domination, $H^2 = \frac{8\pi G}{3}\rho_m$ with $\rho_m \propto a^{-3}$, so H^2 tracks the matter density, not the fold coordinate. The relation $H^2 \sim e^{-x/\kappa}$ can hold only when vacuum energy dominates the expansion. Hence the fold-clock advances with expansion in exactly two epochs — early inflation and the present dark-energy era — and is dormant through the matter era between them.*

Corollary 2.2 (The map is piecewise; the matter era is a gap). *$x(a)$ is not a single monotone function of a . It has two active branches (inflation, late dark energy) bracketing a dormant interval over which the fold coordinate is effectively frozen while a grows by most of cosmic history. The matter-dominated era is a gap in the fold sequence, not one of its folds. This is why a single global $x(z)$ has resisted construction: the object sought does not exist; a vacuum-epoch fold-clock with a matter-era gap does.*

Corollary 2.2 is corpus-faithful: the folds are boundary and vacuum phenomena (Papers 27, 32), so their clock is naturally silent while ordinary matter sets the expansion.

3 Reduction to a single posit, and its CMB test

Granting the reframing, the residual freedom collapses to one statement.

Posit 3.1 (Vacuum-epoch fold law). The inflation law $H^2 \sim e^{-x/\kappa}$ is the fold-clock’s dynamical content in *both* vacuum epochs, inflation and the present dark-energy era. (The corpus asserts it only for inflation; extending it to the dark-energy epoch is the single residual posit.)

Posit 3.1 has an independent, falsifiable consequence in the inflation sector, where the corpus already commits the slow-roll parameter $\epsilon \sim \kappa = \alpha^{5/4} = 0.00213$.

Proposition 3.2 (CMB consequence). *With $\epsilon \sim \kappa$, the tensor-to-scalar ratio is*

$$r = 16\epsilon = 0.034,$$

under the observational bound $r < 0.06$ (Planck+BK, 95%): consistent. The scalar spectral index is

$$n_s = 1 - 6\epsilon = 0.987,$$

against the measured $n_s = 0.9649 \pm 0.0042$: high by 5.3σ . This is a genuine tension and is recorded as such; the leading-order slow-roll estimate omits the η contribution, but no η from the corpus is supplied to close the gap. The posit survives on r and is under tension on n_s .

4 Near-closure in fold-progress coordinates

Under Posit 3.1, the dark-energy epoch has $\Lambda(x) \sim e^{-x/\kappa}$ in present-critical units (Addendum 345a, Floor 2). The half-fold-to-now interval is then

$$\frac{\Lambda_{\text{half}}}{\Lambda_{\text{now}}} = 2 \implies \Delta x = \kappa \ln 2 \approx 1.5 \times 10^{-3}.$$

The entire observable dark-energy history is a $\sim 10^{-3}$ sliver of fold-progress: near-closure, now expressed quantitatively in the x coordinate. This is consistent with — and sharper than — the $\sim 93\%$ amplitude bound of Addendum 345a (Theorem 2.1), which is the same statement read in the density-amplitude coordinate rather than the exponential x coordinate.

Status and falsifiers

Status. The fold-to-redshift map is *not* closed. It is reduced to (i) a canonical identification among the three fold coordinates (A, B, C) and (ii) Posit 3.1, and it is reframed as vacuum-gated with a dormant matter-era gap (Corollary 2.2). No parameters are fit: $\kappa = \alpha^{5/4}$, the inflation law, and Θ are all fixed upstream; Planck CMB data enter only as the measured target.

Falsifiers. (F1) If a future CMB measurement pins n_s away from ~ 0.987 with η unable to reconcile, Posit 3.1 fails in the inflation sector. (F2) If a dark-energy reconstruction demands fold-clock motion *during* matter domination (evolution where Proposition 2.1 forbids it), vacuum gating fails. (F3) If a canonical reduction A=B=C is ever derived, Proposition 1.1 is superseded and the map upgrades from reduced to closed.

Data sources. Planck 2018 inflation constraints ($n_s = 0.9649 \pm 0.0042$, $r < 0.06$ with BICEP/Keck).