

Addendum 345a: Inflation and Dark Energy as One Fold Cascade

The DESI confrontation, the golden-ratio refutation,
and the near-closure bound

(Addendum to Paper 32)

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Abstract

Paper 32 derives the static dark-energy residual $\Lambda_0 = 1 - \pi^2/32 \approx 0.6916$ and its fold-progress extension $\Lambda(k) = \Lambda_0 \cdot 4/C(k)$. This addendum makes one synthesis and runs one confrontation. The synthesis: the fold cascade $B^4 \rightarrow S^3 \rightarrow S^2 \rightarrow S^1 \rightarrow S^0$, whose dark-energy density runs from $\Lambda = \infty$ at the pre-fold stage to Λ_0 at completion, is a *single declining engine*; inflation is its violent early end and dark energy its exhausted tail. This unifies three previously separate corpus conjectures (Paper 20 §7.2, Paper 27 §6, Paper 5). The confrontation: against DESI DR2 (2025), which finds dark energy weakening today ($w_0 = -0.821 \pm 0.065$, $w_a = -0.76$, time variation preferred at $2.8\text{--}4.2\sigma$), we establish three quantitative results. (i) **Direction**: the robust DESI signal — dark energy modestly higher in the recent past, declining now — is the fold engine relaxing onto its floor. (ii) **Magnitude bound**: the integer fold step is a factor of 2 (+100%), but the observed density swing is only +7%; the step overshoots by $\sim 14\times$, so the mechanism is viable *only* if “now” sits $\sim 93\%$ through the final fold — a derived near-closure bound. (iii) **Golden-ratio refutation**: a golden self-similar fold rate, suggested by the nesting-sphere structure, is *excluded*: the breath’s rotation number $\rho = (\pi\alpha^{-1}) \bmod 1 = 0.5122$ is near-rational (continued fraction with large partial quotients, near the tritone $\frac{1}{2}$), the maximal opposite of the noble number; the corpus’s native self-similar ratio is 2, and φ belongs to the E8 sector (Addenda 94/95), not the cascade. We also locate “now” away from full closure: $w_0 > -1$ at $\gtrsim 3\sigma$ forces $x_{\text{now}} < 1$. The fraction-versus-density unit reconciliation is *closed* (§4): the dynamic Λ is the dark-energy density in present-critical units, not a fraction, and the static residual is the present fraction; the two coincide today by construction. Two floors remain open and are not crossed: the fold-to-redshift map (Conjecture 32.2) and the deep-time join to the primordial epoch. All numerical claims are pinned by `verify_P345a.py` (19/19).

1 The synthesis: one declining engine

Three corpus statements describe the same object from different ends. Paper 20 §7.2 conjectures that “the breathing dynamics $C \circ P = I$ at early times drives inflation.” Paper 27 §6 reads the cosmic arc as the descent from Ω , with dark energy “the overhead cost of maintaining consensus across expanding space” and the Big Bang as the first disagreement. Paper 5 lists “inflation from geometric oscillation dynamics.” Paper 32’s fold-progress table closes the loop quantitatively: the dark-energy density runs

$$\Lambda(k): \quad \infty \longrightarrow \infty \longrightarrow 2\Lambda_0 \longrightarrow 2\Lambda_0 \longrightarrow \Lambda_0 \quad (k = 0, 1, 2, 3, 4),$$

from a divergent pre-fold value to the static floor Λ_0 .

Proposition 1.1 (Cascade unification). *Inflation and dark energy are the two ends of the single fold cascade $B^4 \rightarrow S^3 \rightarrow S^2 \rightarrow S^1 \rightarrow S^0$: the early, divergent stages ($\Lambda \gg \Lambda_0$) are the inflationary epoch, and the final stage’s relaxation onto Λ_0 is the present dark-energy era. No new object is introduced beyond the Paper 32 §5 fold-collapse machinery; the proposition names a single engine where the corpus previously carried three separate conjectures.*

The content of Proposition 1.1 is qualitative and inherits the conjecture status of its three sources. What follows tests its one sharp, observable consequence — that the engine is *declining* now — against current data.

2 The DESI confrontation

DESI DR2 (CMB+DESI+SN, w_0w_a CDM) reports $w_0 = -0.821 \pm 0.065$ and $w_a = -0.76^{+0.31}_{-0.26}$, a preference for time-varying dark energy at $2.8\text{--}4.2\sigma$, with $w > -1$ today and $w < -1$ in the recent past. The dark-energy density history for $w(a) = w_0 + w_a(1 - a)$ is

$$\frac{\rho_{\text{DE}}(a)}{\rho_{\text{DE},0}} = a^{-3(1+w_0+w_a)} \exp[-3w_a(1-a)]. \quad (1)$$

(i) Direction. Equation (1) with the DESI central values has ρ_{DE} modestly higher in the recent past and declining toward today. This is exactly the fold engine relaxing onto its floor. The apparent rise on the far side — ρ_{DE} small in the deep past — is the linear- $w(a)$ extrapolation beyond the range DESI constrains ($z \lesssim 2$) and is the least robust part of the fit; the robust statement is the recent decline, which is fold-faithful.

(ii) The magnitude bound and near-closure. The integer cascade changes Λ by a factor of two across an active fold ($2\Lambda_0 \rightarrow \Lambda_0$, i.e. +100%). Equation (1) gives a gentle, non-monotone density bump peaking at only +7.1% near $z \approx 0.31$. The fold step therefore overshoots the observed swing by $\sim 14\times$.

Theorem 2.1 (Near-closure bound). *If the present dark-energy evolution is the tail of the final fold, then the visible residual amplitude ($\sim 7\%$) as a fraction of a full fold step (100%) places “now” at*

$$1 - \frac{\Delta\rho_{\text{obs}}}{\Delta\rho_{\text{fold}}} \approx 1 - \frac{0.071}{1.00} \approx 0.93,$$

i.e. $\sim 93\%$ through the final fold, with only its last $\sim 7\%$ of amplitude still unfolding. The factor-of-two fold step is reconciled with the small observed evolution only under near-closure; this is not an assumption but a consequence of the magnitude mismatch.

(iii) The golden-ratio refutation. The fold sequence is literally nesting spheres, and the golden ratio is the canonical fixed point of self-similar nesting, so a golden fold rate is the natural conjecture. It is excluded by the corpus’s own clock. The breath rotation number is

$$\rho = (\pi \alpha^{-1}) \bmod 1 = 0.5122452, \quad \rho = [0; 1, 1, 19, 1, 10, 1, 12, 8, 4, \dots].$$

The large partial quotients (19, $\dots$, 129) make ρ a *near-rational* sitting beside the tritone $\frac{1}{2}$ (Paper 37: “each breath advances the fibre by almost exactly a half-turn”). The golden ratio is $\varphi = [0; 1, 1, 1, 1, \dots]$, all ones — the noble, maximally irrational number. The breath is its opposite in character. The corpus’s native self-similar ratio is therefore 2 (halving, octave, tritone), and a golden ratio step (+61.8%) still overshoots the observed +7% by $\sim 9\times$. Where φ does live in the corpus — the E8 sector (Addenda 94, 95) — is structurally disjoint from the cosmological cascade.

3 The location of “now”

At full closure the fold-progress function f (with $\Lambda = \Lambda_0 f$, $f(1) = 1$, $f > 1$ for $x < 1$) attains its minimum, so $f'(1) = 0$ and $w = -1$ exactly: no evolution. DESI measures $w_0 = -0.821 \neq -1$ at $2.8\text{--}4.2\sigma$, a statement about the *shape* of $w(z)$ and more robust than the absolute density.

Corollary 3.1 (Now is not closure). *$w_0 > -1$ at $\gtrsim 3\sigma$ implies $f'(x_{\text{now}}) \neq 0$, hence $x_{\text{now}} < 1$: the final fold is in progress, not complete. The static identification of “now” with full closure ($k = 4$, the boundary condition that fixes the normalisation $4/C(k)$) is therefore a definitional placement, not a derived location, and is contradicted at the DESI significance. The static floor Λ_0 is the future asymptote the engine is relaxing onto, not the present value.*

Corollary 3.1 and Theorem 2.1 agree: “now” is near, but short of, full closure — the descending tail of the last fold.

4 Closing Floor 2: the fraction–density reconciliation

The static and dynamic uses of Λ appear to conflict: the corner residual $\Lambda_0 \approx 0.69$ is a volume *fraction* (bounded by 1 by construction) identified with $\Omega_{\Lambda,0}$, yet the dynamic $\Lambda(2) = 2\Lambda_0 \approx 1.383$ exceeds 1. The resolution is that the two are different cosmological quantities, and the dynamic formula is already the correct one.

Proposition 4.1 (Fraction–density reconciliation). *Write $\rho_{\text{crit},0} = 3H_0^2/8\pi G$. Then:*

1. *the static residual is the present density fraction, $\Lambda_0 = \Omega_{\Lambda,0} = \rho_{\Lambda,0}/\rho_{\text{crit},0} \leq 1$;*
2. *the fold factor, normalised to the present, is the density evolution,*

$$\frac{\rho_{\Lambda}(k)}{\rho_{\Lambda,0}} = \frac{4/C(k)}{4/C(4)} = \frac{4}{C(k)}, \quad C(4) = 4;$$

3. *hence the dynamic quantity is the dark-energy density in present-critical units, $\Lambda(k) = \Lambda_0 \cdot 4/C(k) = \rho_{\Lambda}(k)/\rho_{\text{crit},0}$, which may exceed 1 in the past without being a fraction.*

The boundary condition $\Lambda(4) = \Lambda_0$ is then not an extra assumption but the identity “density in present units equals the present fraction,” holding because $\rho_{\text{crit}}(\text{now}) = \rho_{\text{crit},0}$.

Corollary 4.2 (The fraction never exceeds unity). *For any matter density $\rho_m > 0$ the epoch fraction is $\Omega_{\Lambda}(k) = \rho_{\Lambda}(k)/[\rho_{\Lambda}(k) + \rho_m + \dots] \leq 1$. Numerically, the half-fold density $2\Lambda_0 = 1.383$ in present units gives $\Omega_{\Lambda} = 1.383/(1.383 + \Omega_m(1+z)^3)$, which equals 0.815 at the $z = 0$ floor and is smaller at the earlier, denser epoch where the half-fold actually sits. The > 1 values of $\Lambda(k)$ are densities; the corresponding fractions are bounded automatically.*

Remark 4.3 (Consequence for the confrontation). The reconciliation retroactively well-poses §2: the DESI quantity $\rho_{\text{DE}}(z)/\rho_{\text{DE},0}$ and the corpus factor $4/C(k)$ are the same kind of object — dark-energy density normalised to the present — so §2 was density-to-density throughout, not a category error. Floor 2 is closed; it required no new object, only the correct identification of the dynamic Λ as a density rather than a fraction.

5 Remaining floors and falsifiers

This addendum bounds the dynamic model and closes one of its floors (§4). Two floors remain, recorded but not crossed.

Remark 5.1 (Floor 1: the fold-to-redshift map, Conjecture 32.2). No closed-form $x \leftrightarrow z$ map exists, so no quantitative $\Lambda(z)$ curve can be laid over DESI’s redshift bins. Every result here is a *direction* or a *magnitude bound*; the verifier performs no fit. Closing the map is what would upgrade this addendum to a confrontation paper.

Remark 5.2 (Floor 2: the deep-time join). The pre-fold divergence $\Lambda(k \leq 1) = \infty$ joining to a physical $\sim 10^{-32}$ s inflationary epoch is the $x \leftrightarrow z$ map at its extreme. The *direction* is corpus-faithful (Proposition 1.1, Papers 20/27); the *magnitude* is unwritten.

Falsifiers. The picture is falsifiable on the data already arriving. (F1) Near-closure predicts dark energy continues weakening toward $\Lambda_0 \approx 0.6916$ and then *halts*: $w \rightarrow -1$ from above, never crossing below. If future DESI/SN data show w continuing past -1 or oscillating, the fold tail is refuted. (F2) The robust signal must remain a late-time *decline*; if deep- z reconstructions firmly establish a genuine rise of ρ_{DE} from near zero (a true overshoot), then a monotone fold descent is insufficient and an oscillatory (ring-down) completion is required — new structure beyond this addendum. (F3) The golden refutation is falsified only by a re-derivation of the breath rotation number; $\rho = 0.5122$ is pinned by $\pi\alpha^{-1}$.

Status

Synthesis (Proposition 1.1) is conjecture-tier, inheriting Papers 20/27/5. The DESI results (§2, Theorem 2.1, Corollary 3.1) and the golden refutation are quantitative and verifier- pinned. The three floors are open. No parameters are fit to cosmological data: the fold step, the breath rotation number, and Λ_0 are all fixed upstream; the DESI values enter only as the measured target. Verifier: `verify_P345a.py`, 19/19 pass.

Data sources. DESI DR2 dark-energy constraints (2025); Planck 2018 Ω_Λ . See the DESI DR2 results release and the DESI DR2 BAO dynamical-dark-energy analyses.