

Addendum 343 — Blast-Radius Audit of the A337 T_{3B} Correction: Re-grounding the Monster Cluster and the ToENet Luminosity Flag

TOE Corpus

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Outcome: AUDIT. **Addresses:** the blast radius of A337’s retraction of “ $T_{3B} = j^{1/3}$ as a Monster 3B McKay-Thompson moonshine value.” **Verifier:** `Lumen/corpus/addenda/verify/verify_P343.py` (20 checks, 20 PASS / 0 FAIL; A337 facts recomputed from scratch with `mpmath` at `dps= 40`, plus a machine-checked classification manifest). **Net:** no computed fact is retracted (*no* REFUTE); every affected location either SURVIVES under re-attribution or is a framing REVISE. The ToENet $\{12^k\}$ -McKay-Thompson luminosity grounding does *not* survive and is flagged for a dedicated re-derivation before the build proceeds.

Abstract

Addendum 337 retracted the identification of $j(\tau)^{1/3}$ with the Monster’s class-3B McKay-Thompson series: $j^{1/3} = E_4/\eta^8$ is the E8/G2 cube-root function of A160/A161 (value 12 at $\tau = i$), whereas the canonical 3B hauptmodul is $T_{3B}^{\text{std}} = (\eta/\eta_3)^{12} + 12$ (value 535.592 at $\tau = i$). The retracted claim and its corollaries thread through the Monster/exceptional cluster (A160–A182) and into one build target (ToENet’s canonical/dark luminosity flag, PRIORITIES §0a). This addendum maps the full blast radius. We read every candidate location, classify each SURVIVES / REVISE / REFUTE, and give a one-line corrected statement. The audit’s central finding is *surgical*: **nothing is refuted**. Every E8/G2 root-geometry result (A160, A165–A169, A182) stands untouched; the perfect-cube CM uniqueness of $\tau = i$ (A172) is a statement about the function $j^{1/3}$ and survives under re-attribution; the honest Ramanujan-artefact flag for $T_{2B}(\tau_{-163}) = -24$ (A177/A181) stands. What requires revision is the *Monster-moonshine framing* layered on top of the (correct) cube-root number 12: A161’s closing remark, A163, A164, A170, A172, A177, A181 each attach the number to the wrong provenance and now carry an erratum. The most consequential framing correction is A170’s claim that the $G_2 \rightarrow$ Monster chain *reaches* the Monster: per A338 the corpus reaches only E8/Leech, and the Ogg/supersingular Monster contact is left to mainstream moonshine. Finally, the ToENet luminosity criterion — “canonical luminosity = a McKay-Thompson series value landing in $S = \{12^k\}$ ” — had exactly one example, $T_{3B}(i) = 12$, now re-attributed to E8/G2 root geometry. No genuine Monster McKay-Thompson value lands in $\{12^k\}$ ($T_{1A}(i) = 984 \notin \{12^k\}$; $T_{3B}^{\text{std}}(i) = 535.6 \notin \{12^k\}$), so the moonshine grounding of the luminosity flag has no surviving instance. We do not paper over this: the luminosity flag is flagged for a fresh basis.

1 Method

A337 corrects an *attribution*, not a computation. The audit therefore classifies each downstream location by whether the underlying number/lemma is true under its *correct* provenance:

- SURVIVES (re-attribute): the statement is true under the corrected attribution (the 12 as E8/G2 root geometry; $T_{1A}(i) = 984$; the E8 dimension 248 as $[q^1]j^{1/3}$). No retraction of the math; note the re-attribution.

- **REVISE**: the statement is partly right but its Monster-moonshine framing must change (e.g. “unique $\{12^k\}$ McKay-Thompson hit” $\rightarrow$ “ $j^{1/3}(i) = 12$, an E8/G2 root-geometry value, not a Monster series value”).
- **REFUTE**: the statement depends essentially on the false identification and does not survive.

The verifier `verify_P343.py` recomputes the A337 facts from scratch and encodes the classification as a machine-checked manifest (checks 11–20). The audit’s strongest single result is that the **REFUTE** column is *empty*: the math is never wrong; only the Monster-moonshine attribution is.

2 The classification table

Locations are listed by addendum (or build file). “Asserts” marks whether the location states one of: (i) “ $T_{3B} = j^{1/3}$ ”; (ii) “ $T_{3B}(i) = 12$ is a Monster McKay-Thompson value”; (iii) “the unique $S \cap \{12^k\}$ McKay-Thompson hit”; (iv) “canonical luminosity = a McKay-Thompson value in $\{12^k\}$ ”; (v) “the corpus chain reaches the Monster.”

Location	Verdict	Corrected one-line statement
A160 Monodromy-Weyl A_2	SURVIVES	Pure cube-root geometry: monodromy of $j^{1/3}$ is $\mathbb{Z}/3\mathbb{Z} = \text{Weyl}(A_2)$; $12 = J_{\text{short}}$ at $\tau = i$. No Monster claim; untouched.
A161 E8Monster-Bridge	REVISE	Theorems A–C (the $j^{1/3} = E_4/\eta^8 = q^{-1/3}(1 + 248q + \dots)$ q -expansion, McKay’s $248 = \dim E_8$, the $G_2 \leftrightarrow E_8$ chain) SURVIVES as E8/G2 root geometry; the <i>closing remark</i> “ $j^{1/3}$ is the McKay-Thompson series for class 3B, placing G_2 in contact with the Monster” is the retracted conflation — REVISE to “ $j^{1/3}$ is the E8/G2 cube-root function, not the Monster 3B series (A337).”
A163 Monster-McKayThompson	REVISE	Already errata’d by A337. The $j^{1/3}/T_{1A}/T_{2A}/T_{2B}/T_{3A}$ computations stand; the “ $T_{3B}(i) = 12$ is a Monster 3B value” framing is retracted.
A164 StableNull-Grid	REVISE	$\Delta(i) = \eta(i)^{24}$ and $j(i) = J_{\text{short}}^3 = 1728$ SURVIVES (independently derived in A158 from G_2/F_4 root geometry); the provenance line “ $j(i) = J_{\text{short}}^3$ was <i>proved using</i> the McKay-Thompson identification $T_{3B}(i) = J_{\text{short}}$ (A163)” is REVISE to “ $j(i) = J_{\text{short}}^3$ follows from A158’s root geometry; the $j^{1/3}(i) = J_{\text{short}}$ contact is E8/G2, not Monster 3B.”
A165 BF4_36, A166 E6Jordan, A168 E8_OM, A169 Leech, A182 RootComplement	SURVIVES	Every use of $12 = J_{\text{short}}$ is E8/G2/ F_4/E_6 root geometry (Coxeter numbers, root counts, $E_4(i) = J_{\text{short}} \eta(i)^8$, Golay [24, 12, 8] dimension). No Monster McKay-Thompson dependence; untouched. A169’s Leech $\rightarrow$ Monster remark is mainstream moonshine, not a corpus bridge (consistent with A338).

Location	Verdict	Corrected one-line statement
A170 Monster_- ModularBoundary	REVISE	The Moonshine exposition (Conway–Norton, Borcherds, $V^\natural$) SURVIVES as mainstream mathematics; the two load-bearing <i>corpus</i> claims — “ $T_{3B}(i) = j(i)^{1/3} = J_{\text{short}} = 12$ is the unique McKay-Thompson series returning an exact TOE constant” and “the chain from G_2 reaches the Monster” — are REVISE. Corrected: the 12 is the E8/G2 cube-root value, not a McKay-Thompson value; the corpus chain reaches E8/Leech, and the Ogg/supersingular Monster contact is real but unearned, left to mainstream moonshine (A337 §5, A338).
A172 CM_Uniqueness	REVISE	The result “ $\tau = i$ is the unique class-number-one CM point with $j(\tau_D) \in \{12^k\}$ ” SURVIVES — it is a statement about the <i>function</i> j (perfect-cube CM theory), independent of moonshine. The framing line “recovers $J_{\text{short}} = j(i)^{1/3}$ via the $3B$ McKay-Thompson series” is REVISE to “via the E8/G2 cube-root function $j^{1/3} = E_4/\eta^8$ (A337).”
A177 McKay- Thompson_CM_Scan	REVISE	Already errata’d by A337. The 45-evaluation scan stands as arithmetic; $T_{1A}(i) = 984$ and the $T_{2B}(\tau_{-163}) = -24$ Ramanujan-artefact flag survive; the “ $(T_{3B}, \tau_i) = 12$ is the unique $S \cap \{12^k\}$ Monster McKay-Thompson hit” (Prop. 5.1) is retracted <i>as a moonshine statement</i> .
A181 T2B_Ramanujan	REVISE	The result “ $T_{2B}(\tau_{-163}) \approx -24$ is a Ramanujan eta-expansion artefact, $-24 = -2J_{\text{short}}$ ” SURVIVES. The provenance “ J_{short} established in P163 ($T_{3B}(i) = 12$)” is REVISE to “ $J_{\text{short}} = 12$ established in P151/P158 as G_2 root geometry; $j^{1/3}(i) = 12$ is the E8/G2 contact, not a Monster $3B$ value.”
A185 G1P171_Char- acterisation	SURVIVES	“Moonshine” appears only as the name of one PSLQ basis (a null result ruling out moonshine-type formulae for G_1); no dependence on the T_{3B} identification. Untouched.
PRIORITIES §0a (ToENet)	REVISE	The $G_2 \rightarrow \text{Monster}$ -chain claim is REVISE (the chain reaches E8/Leech, not the Monster — A338); the luminosity grounding “canonical = a McKay-Thompson value in $\{12^k\}$ ” is REVISE to <i>flagged-for-re-derivation</i> (§3).
prime_survey_toe_msp	SURVIVES	Already updated by A337 (RESOLVED 2026-06-14 note); consistent with this audit. No change.

Verdict tally: SURVIVES $\times 6$ location-groups, REVISE $\times 8$, REFUTE $\times 0$. *No computed fact is retracted.*

3 The ToENet luminosity verdict

PRIORITIES §0a grounds ToENet’s canonical/dark luminosity flag thus:

“The $G_2 \rightarrow \text{Monster}$ chain (P164–P170) + P177 (CM scan, $T_{3B}(i) = 12$ unique $\{12^k\}$ hit) + P181 ... establish what ‘canonical luminosity’ means: a McKay-Thompson series value landing in $S = \{12^k\}$. Dark nodes are everything else.”

The *sole example* of that criterion was $T_{3B}(i) = 12$. Under A337 that 12 is not a McKay-Thompson value — it is $j^{1/3}(i) = E_4(i)/\eta(i)^8$, E8/G2 root geometry. We ask honestly whether the criterion can be recast.

1. **Recast onto genuine McKay-Thompson values in $\{12^k\}$?** The genuine Monster McKay-Thompson values the corpus actually computed at $\tau = i$ are $T_{1A}(i) = 984$ and $T_{3B}^{\text{std}}(i) = 535.592$. Neither is a power of 12 ($984 = 2^3 \cdot 3 \cdot 41$; 535.6 is not even an integer). The verifier confirms the set of genuine McKay-Thompson values in $\{12^k\}$ is *empty* (check 18). The $\{12^k\}$ criterion has no surviving Monster instance. *Does not survive as a McKay-Thompson criterion.*
2. **Recast as an E8/G2 root-geometry criterion?** The number $12 = J_{\text{short}}$ does survive — as the G_2 short-root sector sum / E_6-F_4 Coxeter number / Golay-code dimension. One *could* define “canonical luminosity = contact with an E8/G2 root-geometry constant.” But this is a *different and weaker* criterion: it has no $\tau = i$ uniqueness, no genus-zero modular boundary, and no dark/canonical *dichotomy* (root constants are everywhere in the corpus). It is a candidate, not a drop-in replacement; asserting it now would be inventing a grounding.
3. **Recast onto $T_{1A}(i) = 984$?** $984 \in S$ is a genuine surviving McKay-Thompson value, but it is a single point, not a $\{12^k\}$ -membership rule, and membership in the hand-assembled set S is weak evidence (A337 §3.2 already noted this). Not a basis for a canonical/dark *dichotomy*.

Verdict: NEEDS-FRESH-BASIS. The ToENet luminosity flag’s moonshine grounding does *not* survive A337. Its only example was the re-attributed 12; no genuine Monster McKay-Thompson value lands in $\{12^k\}$; and the nearest survivors (E8/G2 root constants; $T_{1A} = 984$) do not reproduce the canonical/dark dichotomy the criterion was supposed to supply. We do *not* manufacture a replacement here. The luminosity flag requires a dedicated re-derivation probe before the ToENet build proceeds. A live candidate worth that probe — and notably *independent* of moonshine — is the corpus’s own kernel-nullspace semantics (A272: dark = dynamics of the kernel of a closure map; canonical = locked, dark = drifting, A268), which already carries the dark/canonical dichotomy the $\{12^k\}$ criterion was reaching for.

4 Errata applied

A337 placed errata on A163 and A177. This audit (A343) extends the same surgical erratum — a header comment and a visible **Erratum** paragraph after `\maketitle`, bodies preserved as audit trail — to the remaining addenda that assert the retracted Monster framing and did not yet carry one:

A161 (closing-remark conflation), A164 (“proved using T_{3B} ” provenance), A170 (the Monster-reaching chain), A172 (“ $3B$ McKay-Thompson” provenance), A181 (J_{short} -via-P163 provenance).

The build file `Documents/Specs/PRIORITIES.md` §0a is corrected in place: the $G_2 \rightarrow \text{Monster}$ -chain claim now reads “reaches E8/Leech, not the Monster (A338)” and the luminosity grounding now reads “needs a fresh basis (A343).” Addenda A160, A165–A169, A182, A185 and `Science/Primes/prime_survey_toe_map.md` require no change.

5 What I could not fully resolve

Two honest residuals.

- **A replacement luminosity grounding is not derived here.** This audit establishes that the moonshine grounding fails and names a candidate (A272 kernel-nullspace semantics); it does not derive a criterion. That is a separate probe, correctly gated behind this one.
- **The classification reads each addendum’s *stated* claims,** not every transitive citation. The manifest is exhaustive over the grep-identified cluster (A160–A185 plus the two build files); a citation that uses $T_{3B}(i) = 12$ only through an intermediate result already classified REVERSE inherits that verdict without separate listing.

6 Conclusion

The A337 retraction is surgical and its blast radius is contained. No computed fact across the Monster/exceptional cluster is wrong; every affected statement either survives under its correct E8/G2 root-geometry provenance or needs only a framing revision away from Monster moonshine. The one genuine *loss* is at the build layer: ToENet’s canonical/dark luminosity flag was grounded in a single $\{12^k\}$ -McKay-Thompson example that A337 re-attributed, and no genuine Monster McKay-Thompson value replaces it. That grounding is flagged for re-derivation, not patched over. Errata are recorded on A161/A164/A170/A172/A181 (joining the A337 errata on A163/A177); PRIORITIES §0a is corrected; the verifier `verify_P343.py` confirms the A337 facts and the classification at 20/20.

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