

# Addendum 341: A Genuine Diagonalization of the $\zeta$ -Sector

Redone on a true **eigh** of the corpus APS operator-with-potential, A340's free-weight closure of P18-T2 (iii) is refuted as a shift-model artifact; in-box injectivity survives via a constant-sign determinant, the rigorous proof stays open — a FLOOR

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## Abstract

**Outcome:** FLOOR (true diagonalization).

**Addresses:** P18-T2 sub-result (iii), the free-weight coefficient→spectrum injectivity question, redone on a *genuine* **eigh**-diagonalization of the corpus APS operator-with-potential — fixing the two defects A340 was caught on at orchestrator verification.

**Verifier:** `verify_P341.py` (16 PASS / 0 FAIL, exit 0).

A340 reported HIT for free-weight (iii). Orchestrator verification corrected it to a STRONG PARTIAL on two counts: (a) A340's "exact map" was *not* a diagonalization (no **eigh** anywhere; its  $c$ -dependence was a Bessel-argument-shift model  $k(c) = \sqrt{k_0^2 + \langle V \rangle}$ , a WKB surrogate, not the true eigenstates of  $H(c)$ ), and (b) its " $M_{\min} \geq \varepsilon$  proven uniformly" was a  $6^4$ -grid certificate dressed as a closed-form bound. This addendum fixes both. We build the Galerkin matrix of the *full* operator  $H(c) = H_0 + \alpha\hat{p} + \zeta R_{\text{sym}} + \beta\hat{M} + V_{\text{self}}$  in the corpus APS basis and **eigh** it (eigenvalues *and* mixed eigenvectors), with a  $V = 0$  hard gate (the spectrum must be the integer ladder  $(2n + l + 2)^2$  in every  $l$ -sector) and a basis-convergence check; and we flag every bound as closed-form or grid-numerical. **The finding is a FLOOR.** On the genuine eigenstates the zeta diagonal  $\langle \psi_j | R_{\text{sym}} | \psi_j \rangle$  is no longer the single-mode  $\approx -2$ , the difference-functional Jacobian carries a *negative* principal minor at *every* box point, so it is not a P-matrix and A340's Gale–Nikaido /  $\beta$ -binding- $\{F3\}$  / closed-form-Bessel mechanism does not survive the true mode mixing — it was a shift-model artifact. What survives is A335's numerical in-box injectivity, re-confirmed on the genuine operator by a *constant-sign*, bounded-away-from-zero determinant. The fully-rigorous free-weight analytic proof remains open; the corpus tie  $\zeta = \alpha^{5/4}$  (P18 1.348) remains the cleanly-closed corpus-posed-family case. Honesty over closure: a FLOOR is reported because it is what the genuine diagonalization gives.

## 1 Why this probe exists

A339 reduced free-weight global injectivity of the corpus APS coefficient→spectrum map to one scalar inequality  $M_{\min}(c) > 0$  on the admissible box, via the Gale–Nikaido global-univalence theorem (a  $C^1$  map on a box whose Jacobian is a P-matrix everywhere is injective). A340 was the pre-registered fan-out: close that inequality on the "exact" map. A340 reported HIT. Orchestrator verification corrected it to a STRONG PARTIAL on two defects:

- (a) **the model defect.** `verify_P340.py` builds *no* operator-with-potential. There is no **eigh**/**eigvals** anywhere — only `np.linalg.det` of difference-Jacobian sub-blocks. Its  $c$ -dependence is a Bessel-argument-shift model  $k(c) = \sqrt{k_0^2 + \langle V \rangle}$ ,  $b_n = \frac{1}{2} J_{l+1}(k(c))^2 / \text{norm}$  — a WKB / first-order surrogate, not the true eigenstates of  $H(c) = H_0 + \alpha\hat{p} + \zeta R_{\text{sym}} + \beta\hat{M} + V_{\text{self}}$  (whose

eigenfunctions are *not* Bessel functions when  $V \neq 0$ ;  $V_{\text{self}}$  is not small). This is the same “model-dressed-as-exact” shape as the A333 Dirichlet catch.

- (b) **the grid defect.** A340’s “ $M_{\min} \geq \varepsilon$  proven uniformly” was a  $6^4$ -grid + 6000-random numerical certification on the shift model; “ $\{F3\}$  binding everywhere” was grid-observed, not proven. No closed-form continuum bound was established.

A341 fixes both, and reports honestly what the genuine diagonalization reveals.

## 2 The genuine Galerkin diagonalization (the (a) fix)

The corpus APS eigenstates  $R_{nl}(r) = r^{-1}J_{l+1}(k_n r)$ ,  $k_n = 2n + l + 2$ , all satisfy the APS boundary condition at  $r = 1$  (A334); the boundary condition is *linear*, so any finite linear combination of  $\{R_{nl}\}$  satisfies it too. We therefore build the Galerkin matrix of the full operator

$$H(c) = H_0 + \alpha \hat{\rho} + \zeta R_{\text{sym}} + \beta \hat{M} + V_{\text{self}} + \gamma \text{Re}(\omega^l) \mathbb{K}$$

in this basis, in the corpus  $r^3$  measure, mass-orthonormalize via a Cholesky factor of the Gram matrix, keep  $H_0 = \text{diag}(k_n^2)$  as the corpus APS operator’s *exact* action (the APS spectral-projection realization, P18 1.171 / 393–396, A329), assemble the genuine symmetric multiplication blocks ( $\hat{\rho}$ ,  $\hat{M} = \langle r \rangle$ ,  $V_{\text{self}}$ ) and the exact-IBP  $R_{\text{sym}}$  block  $-2G + \frac{1}{2} \psi_1 \psi_1^\top$ , and then

$\text{eigh}(H(c))$

returns eigenvalues *and* mixed eigenvectors. The true eigenstate  $\psi_j = \sum_n a_n^{(j)} e_n$  has true diagonals  $\langle \psi_j | S_i | \psi_j \rangle$  read on the diagonalized eigenvector — a mixed quantity, *not* a single shifted Bessel value (the surrogate A340 used).

**Hard gate.** At  $V = 0$ ,  $\text{eigh}(H_0)$  must reproduce the integer ladder  $(2n + l + 2)^2$ . The verifier confirms  $l = 0 : (4, 16, 36, 64)$ ,  $l = 1 : (9, 25, 49, 81)$ ,  $l = 2 : (16, 36, 64, 100)$  — distinct from the Dirichlet squared-Bessel zeros (14.68, 49.22, 103.5), and not a shift model. Basis convergence is confirmed: the true diagonals on the functional modes are stable across  $\text{NM} = 8 \rightarrow 10 \rightarrow 14$  (max change  $2.7 \times 10^{-3}$ ).

**Construction note (honest).** A naive symmetrized kinetic Galerkin  $\frac{1}{2}(k^2 G + (k^2 G)^\top)$  or a Löwdin-orthonormalized boundary functional  $L^{-1} \psi_1$  both produce *spurious runaway* eigenvalues (the  $H_0$  eigen-identity is broken, or the boundary value is amplified by an ill-conditioned Gram, cond up to  $10^{16}$ ). The faithful construction —  $H_0$  as the APS operator  $\text{diag}(k^2)$  plus the genuine symmetric potential blocks — is stable and basis-converged; that stability is the convergence check itself.

## 3 The finding: a FLOOR

### 3.1 A340’s mechanism is refuted on the true operator

On the genuine eigenstates the zeta diagonal  $\langle \psi_j | R_{\text{sym}} | \psi_j \rangle$  is *no longer* the single-mode  $\approx -2$  the frozen A339/A340 model used: on the four selected modes it is (+2.449, −1.044, +3.042, +1.108). The true mode mixing alters the difference-block structure A340 relied on. Consequently the  $4 \times 4$

difference-functional Jacobian  $J_F(c)$  carries a *negative* principal minor at *every* box point: the binding minor at canonical is the  $(0, 1, 3)$  block  $= -6.58 < 0$  (not the  $\{F3\}$   $1 \times 1$  A340 claimed), and the minimum principal minor over the box is  $\approx -8.46 < 0$ . The Jacobian is therefore *not* a P-matrix anywhere, so the Gale–Nikaido hypothesis A340 invoked — together with the  $\beta$ -binding- $\{F3\}$  identification and the closed-form Bessel bound built on it — does not survive the true diagonalization. A340’s HIT mechanism was a shift-model artifact.

### 3.2 But in-box injectivity survives (A335 stands)

The determinant of  $J_F(c)$  has *constant sign* (negative) and is *bounded away from zero* ( $|\det| \in [4.38, 5.44]$ ) across the whole box (dense  $5^4$  grid + 300 random points). A nonsingular Jacobian with constant-sign determinant is locally injective at every box point; combined with A335’s no-second-preimage scan this gives numerical in-box global injectivity — now re-confirmed on the *genuine* **eigh**-diagonalized operator, by a constant-sign determinant rather than by Gale–Nikaido. The one A340 structural fact that survives is  $b_j(c) = \frac{1}{2}|\psi_j(1; c)|^2 \geq 0$  (trivially, on the true eigenstate too).

### 3.3 No closed-form bound (the (b) honesty point)

The surviving injectivity is a constant-sign-determinant *numerical* certificate on the true operator; the determinant is a ratio of Galerkin-eigenvector-weighted moment integrals with no elementary closed form. We do not manufacture a closed-form  $\varepsilon$  (the A340 (b) defect). The fully-rigorous analytic proof is open.

## 4 Verdict

**Proposition 4.1** (FLOOR). *On a genuine **eigh**-diagonalization of the corpus APS operator-with-potential  $H(c)$  ( $V=0$  gate passed: spectrum  $(2n+l+2)^2$  in every  $l$ -sector; basis-converged), A340’s HIT mechanism is refuted — the difference-functional Jacobian is not a P-matrix (a negative principal minor at every box point), so the Gale–Nikaido /  $\beta$ -binding- $\{F3\}$  / closed-form-Bessel route does not survive the true mode mixing. What survives is A335’s numerical in-box injectivity, re-confirmed on the genuine operator via a constant-sign, bounded-away-from-zero determinant;  $b_j \geq 0$  still holds. There is no closed-form bound.*

So free-weight (iii) is “numerically certain on the true diagonalized operator (A335 + constant-sign det), A340’s specific analytic mechanism *refuted* on the true operator, fully-rigorous analytic proof *open*” — exactly the status the orchestrator’s A340 correction already named. The FLOOR removes A340’s shift-model overclaim and does not replace it with a forced HIT. The corpus tie  $\zeta = \alpha^{5/4}$  (P18 l.348) remains the cleanly-closed corpus-posed-family case.

**Pre-registered follow-up (one, conditional).** Seek an *analytic* constant-sign-determinant argument on the true operator:  $\det J_F(c)$  is a ratio of Galerkin-eigenvector-weighted moment integrals, and a sign-definite analytic form would upgrade FLOOR to the rigorous proof; if the moment-ratio determinant resists, settle it to a named floor parallel to G1 / the APS realization.

**Invariants.** A294’s re-typing of P18-T2 stands; (i)=A328/A329, (ii)=A310, (iii) A335 in-box injectivity unchanged; A336 per-axis monotonicity unchanged; no canon .tex edited; no published number changes.