

Addendum 338 — $J_3(\mathbb{O}) \rightarrow$ Monster Bridge Feasibility: A Single Grounding Pass Under a Hard Kill-Criterion (No Unforced Standard Contact; Coincidence Left to Moonshine)

TOE Corpus

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Outcome: FEASIBILITY (single pass, no fan-out). **Addresses:** whether the corpus's existing $J_3(\mathbb{O})$ (exceptional Jordan algebra) structure lands on a *standard* monstrous-moonshine object — a canonical McKay–Thompson hauptmodul or a genuine Monster head-representation dimension — *unforced* (no free parameter, no bespoke function), governed by a pre-committed kill-criterion. **Verifier:** `Lumen/corpus/addenda/verify/verify_P338.py` (16 checks, 16 PASS / 0 FAIL, recomputed from scratch with `mpmath` at `dps=40`). **Verdict:** NO UNFORCED STANDARD CONTACT. The contact stops at the first honest check; the nearest target requires a bespoke bridge constant ($323 = 17 \cdot 19$), the A177 failure mode. Recorded as a coincidence, left to mainstream moonshine (FLM / Borcherds 1992). *No new TOE bridge is earned.*

Abstract

This addendum runs one grounding-plus-feasibility pass on a deliberately speculative question: does the exceptional Jordan algebra $J_3(\mathbb{O})$ already present in the corpus reach a *standard* monstrous-moonshine object without forcing? The pass is governed by a hard, pre-committed kill-criterion, written precisely because this is the crank-magnet zone (exceptional algebra $\leftrightarrow$ Monster) where a connection is always forceable. We first state, before any value-matching, what $J_3(\mathbb{O})$ concretely produces in the corpus: it is the 27-dimensional exceptional Jordan algebra whose reduced structure group is E_6 (A166, Prop. 1.1), so the concrete number is $\dim J_3(\mathbb{O}) = 27$, the dimension of the fundamental **27** of E_6 ; and the only place the $J_3(\mathbb{O})$ chain touches the Monster is through the Leech lattice Λ_{24} (A169), whose Monster-adjacent number is the minimal-vector count 196560. We then choose, once, the single most plausible standard target: the smallest faithful Monster head-representation dimension 196883. The match is run once: $196560 \neq 196883$, off by $323 = 17 \cdot 19$ — a non-zero bespoke bridge constant, which is exactly the failure mode A337 retracted (a fitted quantity standing in for a missing identity). No corpus $J_3(\mathbb{O})$ -chain number (27, 78, 1728, 196560) equals any Monster head-representation dimension, and the only $J_3(\mathbb{O}) \rightarrow$ modular function the corpus uses, $j^{1/3} = E_4/\eta^8$, was proved by A337 to be the E_8/G_2 cube-root function, not a Monster hauptmodul. We stop at the first honest failure, as the kill-criterion requires. Verdict: no unforced standard contact; the $196560 \sim 196883$ adjacency is the textbook Leech/Monster near-miss, fully explained by mainstream moonshine, and the corpus has *not* earned a $J_3(\mathbb{O})$ -bridge to it.

1 The kill-criterion (pre-committed)

This is a feasibility pass, not a result. Before any computation, the spine of the deliverable is fixed:

1. State the candidate object *derived from* $J_3(\mathbb{O})$ /corpus structure *before* any value-matching — what graded dimension, character, or modular function does $J_3(\mathbb{O})$ concretely produce in the corpus?

2. It must match a *standard* object with *no* free parameter and *no* bespoke function: either (a) a canonical McKay–Thompson Hauptmodul T_g (matching ≥ 3 q -expansion coefficients, or its value at a CM point against the canonical series), or (b) a genuine Monster head-representation dimension (1, 196883, 196884, 21296876, ...). **Explicitly disqualified:** $j^{1/3}$ and any non-Hauptmodul cube-root or bespoke function (the A177 trap); any object requiring a fitted constant.
3. The *first* honest check that fails the standard-object match $\rightarrow$ STOP. Record “no unforced standard contact; coincidence, left to mainstream moonshine.” Iterating past the first failure to find a tuning that works is itself the failure mode this pass exists to prevent.
4. Even a *pass* would be a *consistency* result only (Borcherds 1992 already explained moonshine) — a candidate for proper follow-up, never “the bridge earned.”

The A337 lesson, just established. A337 retracted A163/A177’s claim that $T_{3B} := j^{1/3}$ and that $T_{3B}(i) = 12$ is a Monster 3B moonshine hit. The function $j^{1/3} = E_4/\eta^8$ is the E_8/G_2 cube-root function (it carries $\dim E_8 = 248$ at first order; A160/A161), *not* the canonical Monster 3B McKay–Thompson Hauptmodul $(\eta/\eta_3)^{12} + 12$ (value 535.59 at i , q -expansion $q^{-1} + 0 + 54q + \dots$). The leap to the Monster *specifically* was bespoke and wrong. This pass tests whether *any* unforced standard contact exists, knowing the prior attempt failed exactly by using a bespoke non-Hauptmodul function.

2 The candidate object (stated first)

Proposition 2.1 (What $J_3(\mathbb{O})$ concretely produces in the corpus). *In the corpus, $J_3(\mathbb{O})$ is the 27-dimensional exceptional Jordan algebra of 3×3 Hermitian octonionic matrices, with*

$$\text{Aut}(J_3(\mathbb{O})) = F_4, \quad \text{Str}_0(J_3(\mathbb{O})) = E_6$$

(A166, §1.2 and Prop. 1.1). *The concrete number $J_3(\mathbb{O})$ produces is $\dim J_3(\mathbb{O}) = 27$, identical to the dimension of the fundamental representation **27** of E_6 . The only place this number touches modular geometry in the corpus is A166, Thm. 5.1:*

$$27 \times 64 = 1728 = j(i) = J_{\text{short}}^3. \quad (1)$$

Equation (1) is recomputed from scratch by the verifier (checks 3–4): $j(i) = 1728$ and $27 \cdot 64 = 1728$. So the corpus’s $J_3(\mathbb{O})$ -to-modular contact is the number 27 (and its cube-partner $j(i) = 1728$), not any character of the Monster.

Proposition 2.2 (The chain to the Monster runs only through the Leech). *The corpus reaches the Monster sporadic group $\mathbb{M}$ from $J_3(\mathbb{O})$ along a single documented path (A169):*

$$J_3(\mathbb{O}) \longrightarrow E_6 \longrightarrow \dots \longrightarrow E_8 \longrightarrow \Lambda_{24} \longrightarrow V^{\natural}(\text{FLM}) \longrightarrow \mathbb{M}.$$

The corpus’s Leech-level number adjacent to the Monster is the minimal-vector count $|\text{minvec}(\Lambda_{24})| = 196560$, the q^2 coefficient of the Leech theta series $\Theta_{\Lambda_{24}} = E_4^3 - 720 \Delta$ (A169).

The verifier recomputes 196560 from the theta series (check 5) and its factorisation $196560 = 2^4 \cdot 3^3 \cdot 5 \cdot 7 \cdot 13$ (check 6). Crucially, the Monster’s head-representation dimensions come from the *graded structure* of $V^{\natural}$ (Borcherds 1992), not from $J_3(\mathbb{O})$; the $J_3(\mathbb{O})$ chain supplies the Leech only as a lattice *input* to the FLM construction. The candidate object for a $J_3(\mathbb{O}) \rightarrow$ Monster contact is therefore the Leech number 196560.

3 The single standard target

We choose, once, the single most plausible standard-moonshine target: the smallest faithful Monster head-representation dimension,

$$196883, \tag{2}$$

together with McKay’s identity $[q^1]j = 196884 = 1 + 196883$ (the q^1 coefficient of the $V^\natural$ partition function j ; verifier checks 7–8). This is the most plausible candidate for an *unforced* contact for three reasons: (i) 196560 is a genuine corpus-derived $J_3(\mathbb{O})$ /Leech number, not a bespoke function; (ii) 196883 is a genuine Monster head-rep dimension, the kill-criterion’s allowed target; (iii) the two differ by only $\sim 0.16\%$, the textbook near-miss that A169 §6.2 already flags ($196883 - 196560 = 323 = 17 \cdot 19$). If the corpus contact were unforced, 196560 would simply *equal* a Monster head-dimension under canonical normalization.

4 The single match, and the kill-criterion verdict

Proposition 4.1 (The standard-object match, run once). *The corpus $J_3(\mathbb{O})$ /Leech number 196560 is not equal to any Monster head-representation dimension:*

$$196560 \notin \{1, 196883, 21296876, 842609326, \dots\}. \tag{3}$$

The nearest target, 196883, is reached only by adding a non-zero bespoke bridge constant:

$$196883 - 196560 = 323 = 17 \times 19 \neq 0. \tag{4}$$

*More generally, no corpus $J_3(\mathbb{O})$ -chain number $\{27, 78, 1728, 196560\}$ (the **27** dimension, $\dim E_6 = 78$, $j(i)$, and the Leech minimal vectors) equals any Monster head-representation dimension.*

Proof. Direct comparison against the canonical Monster head-dimension list, recomputed where modular (verifier S3, checks 9–11). $196560 \neq 196883$; the gap is $323 = 17 \cdot 19$, a non-zero integer. The set $\{27, 78, 1728, 196560\}$ has empty intersection with the head-dimension list. $\square$

First honest check $\rightarrow$ stop. Check 9 (the $196560 \notin$ head-dims test) is the *first* standard-object match in the pass, and it fails: there is no exact, unforced hit. Per the kill-criterion we stop here. We do *not* iterate to a variant — we do not try a different CM point, a different Leech shell, a rescaling, or a constructed function that would close the 323. The non-zero gap 323 is precisely the A177 failure mode: a fitted quantity standing in for a missing identity. The kill-criterion disqualifies it (check 10).

Remark 4.2 (The A177 trap is closed, not re-entered). The only $J_3(\mathbb{O}) \rightarrow$ modular function the corpus actually uses is $j^{1/3} = E_4/\eta^8$ (A161), which A337 proved is the E_8/G_2 cube-root function ($q^{-1/3}(1 + 248q + \dots)$, carrying $\dim E_8 = 248$), not a Monster hauptmodul. Its value $j^{1/3}(i) = 12$ is the G_2 short-root sum J_{short} , and 12 is not a Monster head-representation dimension either (verifier checks 12–13). The trap that produced the A177 error is explicitly avoided, not re-entered under a new name.

Remark 4.3 (What *is* true is mainstream, not a $J_3(\mathbb{O})$ result). The partition function of $V^\natural$ is $j(\tau)$, so $j(i) = 1728$ at the corpus CM point (verifier check 14). This is FLM/Borcherds mainstream moonshine, true regardless of $J_3(\mathbb{O})$. The adjacency $196560 \sim 196883$ is real — both numbers live inside the degree-one graded piece $\dim V_1^\natural = 196884 = 1 + 196883$, and $196884 - 196560 = 324$ (check 15) — but it is the standard Leech/Monster near-miss explained by the moonshine VOA, not an unforced $J_3(\mathbb{O})$ identity. The corpus supplies the Leech as a lattice input; it does not supply the Monster head-dimensions.

Corollary 4.4 (Kill-criterion verdict). *There is no unforced standard contact between the corpus’s $J_3(\mathbb{O})$ structure and a canonical monstrous-moonshine object. The candidate object ($J_3(\mathbb{O}) \rightarrow \dim 27$; chain-to-Monster number = Leech 196560) does not match the single standard target (196883) with canonical normalization and no free parameter; the contact requires a bespoke bridge constant $323 = 17 \cdot 19$, which the kill-criterion disqualifies. The result is a coincidence, left to mainstream moonshine (FLM; Borchers 1992). No new TOE/ $J_3(\mathbb{O})$ bridge to the Monster is earned.*

5 Conclusion

Asked whether the corpus’s existing $J_3(\mathbb{O})$ structure lands on a standard monstrous-moonshine object unforced, the honest answer is no. The concrete $J_3(\mathbb{O})$ number in the corpus is 27 (the E_6 fundamental, via $\text{Str}_0(J_3(\mathbb{O})) = E_6$), and the only number the $J_3(\mathbb{O})$ chain carries toward the Monster is the Leech minimal-vector count 196560 (A169). The single most plausible standard target, the smallest faithful Monster head-representation dimension 196883, is a near-miss, not a hit: it differs from 196560 by the bespoke constant $323 = 17 \cdot 19$. No corpus $J_3(\mathbb{O})$ -chain number equals a Monster head-dimension, and the corpus’s only $J_3(\mathbb{O}) \rightarrow \text{modular function}$ is the E_8/G_2 cube root, not a hauptmodul. We stopped at the first honest failure, as the kill-criterion requires, and did not iterate. This mirrors A337’s posture: the corpus legitimately reaches E_6 , E_8 , and the Leech, but the leap to the Monster *specifically* is not earned — it is a real coincidence, fully explained by mainstream moonshine.

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