

Addendum 337 — Correction of the Monster 3B McKay-Thompson Mismatch: $j^{1/3}$ is the E8/G2 Cube-Root Function, not the 3B Series

TOE Corpus

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Outcome: CORRECTION. **Addresses:** the $T_{3B} := j^{1/3}$ mislabel in P163, P170, P172, and A177 (the 45-evaluation CM scan). **Verifier:** `Lumen/corpus/addenda/verify/verify_P337.py` (18 checks, 18 PASS / 0 FAIL, recomputed from scratch with `mpmath` at `dps=40`). **Net:** the “12 = Monster 3B moonshine hit” attribution is retracted; the $12 = J_{\text{short}}$ fact and the correctly-computed $T_{1A}/T_{2A}/T_{2B}/T_{3A}$ values survive.

Abstract

Addendum 163 and Addendum 177 defined the Monster’s class-3B McKay-Thompson series as $T_{3B}(\tau) := j(\tau)^{1/3}$ (the real cube root) and reported $T_{3B}(i) = 1728^{1/3} = 12 = J_{\text{short}}$ as a Monster 3B *moonshine* hit, the apex of a 45-evaluation CM scan. This labeling conflates two distinct functions both informally called “the cube root” or “3B”: (1) $j^{1/3} = E_4/\eta^8$, the E8/G2 cube-root function of A160/A161, whose value at $\tau = i$ is genuinely 12; and (2) the canonical Monster 3B McKay-Thompson *hauptmodul* $T_{3B}^{\text{std}}(\tau) = (\eta(\tau)/\eta(3\tau))^{12} + 12$, whose value at $\tau = i$ is 535.592230826..., not 12. We recompute both from scratch at 40 decimal digits: $j^{1/3}(i) = 12.000$ exactly, $T_{3B}^{\text{std}}(i) = 535.592...$, and the two disagree by 523.592. Their q -expansions are structurally different: $j^{1/3} = E_4/\eta^8 = q^{-1/3}(1 + 248q + \dots)$ (the E8 adjoint dimension 248), whereas $T_{3B}^{\text{std}} = q^{-1} + 0 + 54q + \dots$ (integer powers, the Conway–Norton 3B coefficient 54). The “12 = Monster 3B moonshine” attribution is therefore *retracted*. This correction is surgical: the $12 = J_{\text{short}}$ fact stands intact under its correct provenance as E8/G2 root geometry (A160/A161, P158); the correctly-computed $T_{1A}(i) = j(i) - 744 = 984$ and the standard-series $T_{2A}/T_{2B}/T_{3A}$ values stand; the perfect-cube CM lemma stands; and the honest A177 flag that $T_{2B}(\tau_{-163}) = -24$ is a Ramanujan eta artefact stands. Only the Monster-3B-moonshine attribution of the cube-root values is withdrawn.

1 The error

P163 (§2.5, §1 below) and A177 (§2.6, Table `tab:main`) introduced

$$T_{3B}(\tau) \stackrel{?}{=} j(\tau)^{1/3} \quad (\text{real cube root}), \tag{1}$$

asserted this to be the McKay-Thompson series of the Monster’s 3B conjugacy class, and reported

$$T_{3B}(i) = j(i)^{1/3} = 1728^{1/3} = 12 = J_{\text{short}} \tag{2}$$

as a Monster 3B *moonshine* hit — the unique element of $S \cap \{12^k : k \geq 1\}$ across A177’s 45 evaluations.

Equation (1) is not the Monster 3B McKay-Thompson series. It is the E8/G2 cube-root function

$$j^{1/3} = \frac{E_4}{\eta^8}, \tag{3}$$

which appears in the corpus as the Monodromy–Weyl- A_2 object of A160 and the E8–Monster bridge of A161, and whose value 12 at $\tau = i$ is the G_2/F_4 short-root sector sum J_{short} of P151/P158 ($1728^{1/3} = 12$ via root-system arithmetic). The McKay–Thompson series of the actual 3B class is a different function entirely (§4). Equation (2) attaches the correct number (12) to the wrong provenance (Monster moonshine rather than exceptional-Lie root geometry). Fixing the provenance is the whole of this correction.

2 What is retracted

The following claims rested on the mislabel (1) and are *retracted as moonshine statements*:

1. The identification “ $T_{3B} = j^{1/3}$ ” (P163 §2.5/Thm. 1, A177 §2.6). The function $j^{1/3} = E_4/\eta^8$ is not the Monster 3B McKay–Thompson series.
2. The claim that the cube-root values $T_{3B}(\tau_D) \in \{12, 0, -15, 20, -32, -96, -960, -5280, -640320\}$ (A177 Table tab:main, T_{3B} column) are Monster 3B McKay–Thompson *moonshine* values. They are the real cube roots $j(\tau_D)^{1/3}$ of a different function.
3. Consequently, the A177 framing “ $T_{3B}(\tau_i) = 12$ is the unique element of $S \cap \{12^k\}$ among Monster McKay–Thompson evaluations” (A177 Prop. 5.1) is *retracted as a moonshine statement*: the 12 is not a value of any Monster 3B series. (As a statement about the cube-root function $j^{1/3}$, the uniqueness of $\tau = i$ among class-number-one CM points where $j \in \{12^k\}$ — the P172 content — is unaffected.)

The survey note that the corpus T_{3B} CM hits “do not survive the standard-series check” is hereby **confirmed and formally recorded**: the standard 3B series check is exactly $T_{3B}^{\text{std}}(i) = 535.592 \neq 12$.

3 What survives

This correction is surgical. It corrects an *attribution*; it does not demolish any computed fact. The following all stand.

1. $j(i)^{1/3} = 12 = J_{\text{short}}$ **is true and meaningful** — as E8/G2 *root geometry*, the value of the cube-root function E_4/η^8 at $\tau = i$ (A160, A161; P158’s $1728^{1/3} = 12$ via G_2/F_4 root sums). The number is re-attributed, not destroyed. The verifier confirms $E_4(i)/\eta(i)^8 = 12$ directly (verify_P337.py S5).
2. $T_{1A}(i) = j(i) - 744 = 984$ is a correct McKay–Thompson value: 1A is by definition $j - 744$, correctly computed. 984 stands as a genuine modular-value fact. (We note in passing that membership in the hand-assembled constant set S is weak evidence; but the *computation* is right and is not retracted.)
3. T_{2A}, T_{2B}, T_{3A} in A177 use correct standard definitions (the $\Gamma_0(2)^+$ and Conway–Norton hauptmoduln, genuine η -quotients). In particular $T_{2A}(i) = 2^9 + 24 = 536$ and $T_{2B}(i) = 2^3 = 8$ via the CM identities $\eta(i)/\eta(2i) = 2^{3/8}$, $\eta(i/2)/\eta(i) = 2^{1/8}$; these are correct.
4. **The** $T_{2B}(\tau_{-163}) = -24$ **result** is correctly flagged by A177 itself (§4.4) as a Ramanujan eta-expansion artefact — the universal leading real correction $24 e^{-\pi\sqrt{|D|}}$ passing the 10^{-10} threshold only at $D = -163$ — not a deep CM identity. That honest flag stands.

5. **The perfect-cube lemma** (A177 Lemma 2.1: $j(\tau_D)$ is a perfect cube for the nine Stark–Heegner discriminants, with cube roots 0, 12, −15, 20, −32, −96, −960, −5280, −640320) is correct classical CM theory and stands. The verifier checks the sample cases $j(\rho) = 0 = 0^3$ and $j(i) = 1728 = 12^3$ (S5). What changes is only the *name* of the function whose values these are: the cube-root function $j^{1/3}$, not the Monster 3B series.

4 E8/G2 cube root vs. Monster 3B: the verified distinction

The two functions are distinguished by both their values and their q -expansions. All numbers below are recomputed from scratch at `dps=40` by `verify_P337.py`.

Values at $\tau = i$.

$$(\text{E8/G2 cube root}) \quad j^{1/3}(i) = \frac{E_4(i)}{\eta(i)^8} = 12.000000 \dots \quad (4)$$

$$(\text{canonical Monster 3B}) \quad T_{3B}^{\text{std}}(i) = \left(\frac{\eta(i)}{\eta(3i)} \right)^{12} + 12 = 523.592230826 \dots + 12 = 535.592230826 \dots \quad (5)$$

The difference is $T_{3B}^{\text{std}}(i) - j^{1/3}(i) = 523.592 \dots \neq 0$. The mislabeled “ $T_{3B} = 12$ ” is precisely the outlier in the 3B comparison: the genuine 3A and 3B series both evaluate to ≈ 536 at $\tau = i$ (A177’s own $T_{3A}(i) \approx 536.985$; $T_{3B}^{\text{std}}(i) = 535.592$), while the cube-root 12 sits two orders of magnitude away because it is secretly $j^{1/3}$.

q -expansions. The cube-root function carries the E8 adjoint dimension at first order:

$$j^{1/3} = \frac{E_4}{\eta^8} = q^{-1/3}(1 + 248q + \dots), \quad (6)$$

with leading exponent $-\frac{1}{3}$ and first coefficient $248 = \dim E_8$ (verifier S3: fitted $a_1 = 248.03$). The canonical 3B hauptmodul has integer powers and the Conway–Norton coefficient 54:

$$T_{3B}^{\text{std}} = q^{-1} + 0 + 54q - 76q^2 - \dots, \quad (7)$$

leading exponent -1 , constant 0, first coefficient 54 (verifier S3: fitted $b_0 = 1.00$, $b_1 = 0.00$, $b_2 = 54.01$). A cube-root function ($q^{-1/3}$, the 248) and an integer-power hauptmodul (q^{-1} , the 54) are not the same series.

Proposition 4.1 (The conflation, made precise). *$j^{1/3}(i) = 12$ and $T_{3B}^{\text{std}}(i) = 535.592 \dots$ are values of two distinct functions. The claim “ $T_{3B}(i) = 12$ is a Monster 3B McKay–Thompson moonshine value” equates them, and is false. The number 12 belongs to the E8/G2 cube-root reading; the Monster 3B series has no TOE-constant contact at $\tau = i$ — nor at the other Stark–Heegner CM points, where T_{3B}^{std} likewise returns $\approx j(\tau_D) - 744$, not the cube roots.*

Remark 4.2 (No E8/G2 result is affected). Every corpus result that rests on the E8/G2 cube-root reading — A160’s monodromy–Weyl- A_2 structure, A161’s E8–Monster bridge, P158’s $1728^{1/3} = 12$ via root-system arithmetic — depends only on $j^{1/3} = E_4/\eta^8$ and its value 12 at $\tau = i$. Those are confirmed by the verifier and are untouched. What is retracted is solely the Monster-3B-moonshine *attribution* that was layered on top of the (correct) cube-root number.

5 The Ogg / supersingular coincidence: left to mainstream moonshine

For completeness we record an adjacent fact that is sometimes invoked in the neighbourhood of these addenda. The set of primes p for which the supersingular elliptic curves in characteristic p are all defined over $\mathbb{F}_p$ coincides with the set of primes dividing the order of the Monster: $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 41, 47, 59, 71\}$ (the fifteen “Ogg primes,” Ogg’s 1975 observation). This is a real and famous coincidence. It is, however, fully explained by mainstream monstrous moonshine (the Conway–Norton conjecture, proved by Borcherds in 1992): the genus-zero property of the moonshine groups $\Gamma_0(p)^+$ for exactly these primes is what both facts express. The corpus has *not* earned a TOE / $J_3(\mathbb{O})$ bridge to this coincidence. We record it as: a real coincidence, no TOE bridge, left to mainstream moonshine.

6 Conclusion

The corpus conflated two functions both informally called “the cube root / 3B”: (1) $j^{1/3} = E_4/\eta^8$, the E8/G2 cube-root function (genuine, A160/A161, value 12 at i), and (2) the canonical Monster 3B McKay–Thompson hauptmodul $(\eta/\eta_3)^{12} + 12$ (value 535.592 at i). The “12 = Monster 3B moonshine hit” claim is the conflation. Fixing it: the 12 belongs to E8/G2 root geometry, and the Monster 3B series has no TOE-constant contact at $\tau = i$ or at the other CM points. No corpus result that rests on the E8/G2 cube-root reading (A160/A161) is affected; only the Monster-3B-moonshine attribution is retracted. The correction is recorded as errata at the head of A163 and A177; their bodies are preserved as an audit trail.

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