

Addendum 326: The Family–Degree Bridge

One posited premise joins the eigenvalue labelling to the coefficient it draws

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** PARTIAL (the bridge is posited as one irreducible premise; two of A325’s three open items are now corpus-forced) — **Addresses:** OI-287-1 (corrected); A325; A287; P20; A304 — **Verifier:** `verify_P326.py` — all PASS

Abstract

A325 located the standing residual of OI-287-1 as a single unstated corpus bridge: P20 indexes a charged-lepton family by its T_{cycle} eigenvalue exponent k , A287 indexes it by the layer-degree of the coefficient it draws, and the two read through the grading $\text{char} = \text{degree} \bmod 3$ disagree by a uniform $+1$. A325 found a unique structural reconciler, $\text{char} = (\text{degree} - 1) \bmod 3$, under the A304-forced forward generator, and flagged three things still in play: the identification of P18’s third layer (“fiber”) with A287’s and the kernel’s “edge”; the electron-as-anchor; and the rule selecting which coefficient a family draws. This addendum makes the bridge as airtight as the corpus allows. It resolves the first two flags, and the sign of the twist, as corpus-forced. The fiber *is* the edge: P05 names “the edge S^1 fibers” carrying the linear term $2\pi x$, the observational share $2.3\% = \pi/\alpha^{-1}$, so P18’s bulk $\rightarrow$ boundary $\rightarrow$ fiber is the degree cycle $3 \rightarrow 2 \rightarrow 1$ with fiber = edge = degree 1, not a separate object. The anchor $k = 0 \leftrightarrow \text{edge} \leftrightarrow \text{degree } 1$ is forced by the electron being the trivial representation (P20) together with $\rho(0) = 0$, which makes the edge the ground of the present degrees $\{1, 2, 3\}$. The relative handedness is a fixed -1 , not a choosable ± 1 , because the layer cycle (degree-decreasing) and the degree grading (character-increasing) both inherit the one A304-forced forward rotation. What does *not* close is the third flag. The family eigenspaces $\psi^{(k)}$ are $\mathbb{Z}_3$ Fourier combinations spread equally over all three layers, so “family k draws a single degree” is a *projection* choice on a spread state; the corpus never derives that the projection is the character-resonant diagonal rather than another $\mathbb{Z}_3$ -equivariant functional. The bridge therefore reduces to one irreducible premise, $G1$: the mass-law coefficient a family draws is the character-resonant layer of its eigenspace. *Given* $G1$, the full bridge $k = (\text{degree} - 1) \bmod 3$ is forced with zero residual and reproduces $(e, \mu, \tau) \leftrightarrow (2, 3, 16)$ end to end through the A287 law. The verdict is PARTIAL: a posited pin, but sharpened from A325’s bare equality to the single premise $G1$, with two of A325’s three open items now corpus-forced. The overlap-diagonal of A299 through A320 stays withdrawn; no wavefunction or overlap magnitude is formed; no published number changes.

1 The Bridge, Stated

Proposition 1.1 (The Family–Degree Bridge). *A charged-lepton family’s T_{cycle} eigenvalue index k equals the ground-relative ρ -degree of the coefficient it draws,*

$$k = (\text{degree} - 1) \bmod 3,$$

with the edge layer (degree 1, the $\rho(0) = 0$ Dirichlet ground) taken as the index origin. This identifies P20’s Family-Origin labelling (the family is the ω^k eigenspace of T_{cycle} , line 201) with

A287’s level assignment (the family draws the coefficient of its layer, lines 60–64): $k = 0$ draws degree 1 (edge, $c = 2$, the anchor), $k = 1$ draws degree 2 (boundary, $c = 3$), $k = 2$ draws degree 3 (bulk, $c = 16$).

This addendum asks how much of Proposition 1.1 the corpus forces, and what single statement remains to be posited. Table 1 sets the joined labelling out.

Family	e	μ	τ
P20 eigenvalue index k (line 201)	0	1	2
Layer drawn (A287, lines 60–64)	edge	boundary	bulk
ρ -degree of that layer	1	2	3
Coefficient c_n	2	3	16
Bridge: (degree $- 1$) mod 3	0	1	2

Table 1: The Family–Degree Bridge joins the two corpus labellings. The eigenvalue index k of P20 equals the ground-relative degree (degree $- 1$) mod 3 of the A287 coefficient, edge as ground. The bottom row reproduces the P20 index exactly.

2 What the Corpus Forces

A325 left three items in play. Two of them, and the sign of the twist, close here.

The fiber is the edge (A325 flag (i), resolved). A325 flagged that P18 §3.5 names the third layer of the cycle “fiber” (bulk $\rightarrow$ boundary $\rightarrow$ fiber), while A287 and the kernel name the degree-1 layer “edge”, and that the bridge depends on whether these are the same layer. They are, and the corpus states it. P05’s figure caption reads “the edge S^1 fibers support observational structure (2.3%)”; its section “The Observational Edge” identifies the linear term $2\pi x$ with “the fibers of observation”; P18 line 127 lists “Fiber S^1 : observational mode (dimension 1)” and line 575 reads $\Delta_{S^1} = \text{Observation}$. All coincide: the fiber is the S^1 Hopf layer, is the degree-1 $2\pi x$ term, is observation, is the share $2.3\% = \pi/\alpha^{-1}$, is the edge. P18’s bulk $\rightarrow$ boundary $\rightarrow$ fiber is therefore the degree cycle $3 \rightarrow 2 \rightarrow 1$ with fiber = edge = degree 1. This is a corpus identity, not a posit.

The anchor is forced (A325 flag (ii), resolved). The electron is the trivial representation: P20 gives it $k = 0$, eigenvalue 1. Because $\rho(0) = 0$, the density has no degree-zero term, so the present degrees are $\{1, 2, 3\}$ and the ground (lowest present layer) is the edge at degree 1. The trivial character is the one whose lowest *present* representative is not the minimal degree — its lowest present term is degree 3, while characters 1, 2 first appear at degrees 1, 2. So the trivial family is the anchor, and the anchor sits at the ground, the edge. The identification $k = 0 \leftrightarrow \text{edge} \leftrightarrow \text{degree 1}$ is forced by P20, $\rho(0) = 0$, and the fiber-equals-edge identity together; it is not a convention.

The handedness sign is forced. The base grading on monomials is $x^d \mapsto \omega^d x^d$, under which the character *increases* with degree. The layer cycle of P18 runs $3 \rightarrow 2 \rightarrow 1$, *decreasing* degree. Both are the one $\mathbb{Z}_3$ acting through the same forward rotation $\varphi \mapsto \varphi + 2\pi/3$, which A304 forces (the backward generator C^2 , root ω^2 , is excluded by the $L(3, 1)$ positive orientation of Papers 06 and 28). One forward layer-cycle step therefore changes the graded character by a uniform -1

(mod 3): the residual is a fixed -1 , not a choosable ± 1 . This is the “one unit of ω ” of A325’s Resolution C, now with its sign pinned to the forced rotation.

Proposition 2.1 (Given the projection premise, the bridge is forced with no residual). *Let G1 be the premise that the mass-law coefficient a family draws is the character-resonant layer of its eigenspace — the layer whose ground-relative grading character equals the family’s eigenvalue index. Given G1, the forced anchor $k = 0 \leftrightarrow$ degree 1, and the forced forward unit step, the assignment of the two non-anchor families is determined uniquely: of the two bijections $\{1, 2, 3\} \rightarrow \{0, 1, 2\}$ that fix the anchor, exactly one advances by the forced unit step, namely $k = (\text{degree} - 1) \bmod 3$. This coincides with the unique reconciler of A325’s convention scan under the A304-forced generator, and the literal grading ($\text{char} = d$) and the naive Dirichlet shift ($\text{char} = d + 1$) both fail. There is no residual freedom once G1 is granted.*

3 What Stays Unforced, and Why the Verdict Is PARTIAL

Remark 3.1 (The eigenspaces are layer-spread; G1 is a projection choice). *The forcing reaches exactly as far as Proposition 2.1 and no further, and it is honest to stop there. The family eigenspaces are not individual layers. In the regular representation that A304 fixes, $\psi^{(k)}$ is the $\mathbb{Z}_3$ Fourier combination $(\text{bulk} + \omega^{-k} \text{boundary} + \omega^{-2k} \text{fiber})/\sqrt{3}$, which carries weight $\frac{1}{3}$ on each of the three layers. “Family k draws a single degree” is therefore a statement about a projection of a layer-spread state, not an identity between a family and a layer. Which functional of the spread eigenspace selects the drawn coefficient is the content of G1. The character-resonant diagonal is the unique $\mathbb{Z}_3$ -equivariant choice that, once adopted, leaves no freedom (Proposition 2.1); but $\mathbb{Z}_3$ -equivariance alone does not pin it — the conjugate equivariant diagonal $k \mapsto -k$ gives $(0, 2, 1)$ and does not reconcile. Selecting the resonant diagonal over the alternatives is the posited premise. It is precisely the route A287 flagged (“layer-projected perturbation of $\hat{O}$, each family’s state drawing a correction proportional to its layer’s coefficient”), and the withdrawn overlap-diagonal of A299 through A320 was the attempt to compute this projection from the operator; it overshot and was withdrawn, which is independent evidence that the projection is not fixed by representation theory without further input. The corpus does not derive G1. The verdict is therefore PARTIAL.*

The pin the corpus must add. One statement upgrades PARTIAL to HIT: *the mass-law coefficient a charged-lepton family draws is the character-resonant layer of its T_{cycle} eigenspace (premise G1).* Given G1, the Family–Degree Bridge $k = (\text{degree} - 1) \bmod 3$ follows as a theorem (Proposition 2.1), and OI-287-1’s why-question closes. The pin is sharper than A325’s: A325 posited the bare equality $k = (\text{degree} - 1) \bmod 3$; A326 shows that equality decomposes into the forced anchor, the forced handedness sign, and the forced no-residual propagation, leaving exactly one premise to posit — the resonant-diagonal projection rule.

End to end. Table 2 carries the bridge through the A287 law. The bridge fixes *which* coefficient each family draws; the A287 law then reproduces the mass ratios at the published 10^{-4} accuracy.

Status. OI-287-1 remains open in its corrected form, but its residual is now reduced to a single irreducible premise. The mechanism is a $\mathbb{Z}_3$ character selection rule (A323); the coefficient set is the density’s own $\mathbb{Z}_3$ character content (A324); the family-to-coefficient labelling reconciles under a unique convention (A325); and the bridge joining the eigenvalue labelling to the drawn degree decomposes, here, into three corpus-forced pieces and one posited premise — the resonant-diagonal

Family	index k	degree $k+1$	coeff c_n	m_n/m_e (pred / meas)
e	0	1	2	1 (anchor)
μ	1	2	3	206.80 / 206.768
τ	2	3	16	3477.4 / 3477.4

Table 2: End-to-end reproduction. The bridge $k \mapsto \text{degree } k+1 \mapsto c_n$ selects the coefficient; the A287 law $m_n/m_e = (\lambda_n/\lambda_1)^{\beta_n}$, $\beta_n = 6\mu_1/\mu_0 - c_n \alpha^{1/4}/\lambda_1$, reproduces the ratios to $\sim 10^{-4}$.

projection of the layer-spread eigenspace. The fiber-equals-edge identification and the electron anchor, which A325 left open, are now corpus-forced. The overlap-diagonal of A299 through A320 stays withdrawn; no wavefunction or overlap magnitude is formed. No published number changes; the A287 lepton law and its 10^{-4} accuracy are untouched.

Verifier. `verify_P326.py`: recomputes $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ and the layer fractions; the layer-degree map and $c_n = (2, 3, 16)$ with $\rho(0) = 0$; the fiber-equals-edge-equals- S^1 -equals-degree-1 identity at share 2.3%; the two labellings; the A304 forward generator and its orientation invariant; the opposite handedness of the layer cycle and the grading as a fixed -1 ; the forced anchor; the convention scan and unique reconciler; the no-residual propagation given $G1$; the layer-spread eigenspaces and the honest unforcedness of $G1$ (the conjugate diagonal fails); the end-to-end reproduction $(e, \mu, \tau) \leftrightarrow (2, 3, 16)$ through the A287 law to m_μ/m_e and m_τ/m_e ; the PARTIAL verdict; and the exact pin. Twenty-five checks, all PASS.