

Addendum 325: The Level–Character Offset

One unstated bridge stands between the family labelling and the coefficient it draws

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** PARTIAL (open reindexing with a named, unique structural reconciler; the corpus must add one pin) — **Addresses:** OI-287-1 (corrected); A323; A324 — **Verifier:** `verify_P325.py` — all PASS

Abstract

A323 found the mechanism behind the family-to-coefficient assignment, a $\mathbb{Z}_3$ character selection rule, and A324 confirmed that the coefficient set $(2, 3, 16)$ is the density’s own $\mathbb{Z}_3$ character content. Both named a residual: a uniform $+1$ character twist between two corpus statements that the corpus never joins. P20 labels the families e, μ, τ by layer-cycle eigenvalue $1, \omega, \omega^2$, that is characters $0, 1, 2$. A287 assigns e, μ, τ to the layers edge, boundary, bulk, that is degrees $1, 2, 3$, and with the grading $\text{char} = \text{degree} \bmod 3$ those layers carry characters $1, 2, 0$, so every family is off from its P20 character by the same $+1$. This addendum asks whether that twist is forced or open. A complete scan over the two free choices, the generator orientation and the anchor of the character ladder, shows that under the forward generator that A304 forces there is a *unique* reconciling convention, $\text{char} = (\text{degree} - 1) \bmod 3$, the edge-anchored ground-relative degree, and that neither the literal grading nor a naive Dirichlet $+1$ shift reconciles. The twist has a named structural origin: the layer cycle ($\text{degrees } 3 \rightarrow 2 \rightarrow 1$, P18 §3.5) and the degree grading ($x^d \mapsto \omega^d x^d$) run with opposite handedness on the degree ladder, so matching them costs one unit of ω (Resolution C), with the edge-as-ground anchoring motivated by $\rho(0) = 0$ (Resolution B) and the trivial-representation-is-anchor identity (Resolution A) necessary but only relabelling. The forcing is not airtight: the corpus never states the bridge fixing that the family index follows the T_{cycle} eigenvalue while the coefficient is drawn by degree. The verdict is PARTIAL, an open reindexing with a named and unique structural reconciler. The corpus must add one pin, $k = (\text{drawn degree} - 1) \bmod 3$, to upgrade to a closure. The overlap-diagonal of A299 through A320 stays withdrawn; no wavefunction or overlap is formed. No published number changes.

1 The Two Labellings and the Twist

The probe joins two statements the corpus makes in different places and never aligns. Table 1 sets them side by side.

The twist is uniform and nonzero. The family that P20 calls character k draws, under A287, the coefficient whose degree carries character $k + 1$. This is the $n = k + 1$ offset A323 named and A324 carried as the mod-3 reindexing. The question P4 must settle is whether that uniform $+1$ is derivable from the corpus or is a genuine open reindexing the corpus must pin.

Family	e	μ	τ
P20 character k (eigenvalue ω^k , line 201)	0	1	2
A287 layer (lines 60–64)	edge	boundary	bulk
A287 degree of drawn coefficient	1	2	3
A287 coefficient c_n	2	3	16
character of that degree (deg mod 3)	1	2	0
twist (A287 character – P20 character, mod 3)	+1	+1	+1

Table 1: The two corpus labellings of the charged leptons, and the uniform +1 character twist between them. P20 indexes a family by its T_{cycle} eigenvalue; A287 indexes it by the layer of the coefficient it draws. Read through the grading char = degree mod 3, the two disagree by the same +1 on every family.

2 The Decisive Convention Scan

Proposition 2.1 (A unique reconciling convention exists under the forced generator, and it is not the literal grading). *The labelling carries two free choices, each touching the corpus. The first is the generator orientation: A304 builds the $\mathbb{Z}_3$ action as its regular representation and shows that the forward generator $C = T_{\text{cycle}}$, carrying root ω , is forced by the positively oriented quotient $L(3, 1)$ of Papers 06 and 28, with the backward generator C^2 , carrying ω^2 , excluded. The second is the anchor of the character ladder relative to the degree ladder, a constant $g_0 \in \{0, 1, 2\}$ added before grading. For a present degree d the assigned character is $\chi(d) = \text{gen} \cdot (d + g_0) \bmod 3$, where $\text{gen} = +1$ is the forward generator and $\text{gen} = -1$ the backward one (conjugation $k \mapsto -k$).*

A reconciliation requires $\chi(\text{degree}_f) = \text{char}_f^{\text{P20}}$ for all three families. Scanning the six conventions, exactly one reconciles, and under the A304-forced $\text{gen} = +1$ it is unique:

$$\boxed{\chi(d) = (d - 1) \bmod 3} \quad (\text{the choice } g_0 = 2 \equiv -1),$$

which sends degrees 1, 2, 3 to characters 0, 1, 2, that is edge to e , boundary to μ , bulk to τ , matching P20 exactly. The literal grading $\chi(d) = d \bmod 3$ ($g_0 = 0$) gives (1, 2, 0) and does not reconcile; it is the twisted labelling. The naive Dirichlet up-shift $\chi(d) = (d + 1) \bmod 3$ ($g_0 = 1$) gives (2, 0, 1) and also fails. Only the edge-anchored ground-relative rule $\chi(d) = (d - 1) \bmod 3$ succeeds. The reconciler is therefore fixed and unique, but it is not the corpus’s literal action.

The reconciling rule has a transparent reading. Because the density satisfies the edge condition $\rho(0) = 0$ and so carries no degree-zero term, its degrees are $\{1, 2, 3\}$, not $\{0, 1, 2\}$; the natural ground is the edge at degree 1, and the ground-relative index $d - 1$ runs 0, 1, 2 over the present layers (Resolution B). The reconciler $\chi(d) = (d - 1)$ is precisely this ground-relative index. The trivial representation, character 0, is the one with no degree-matched term in ρ , its lowest representative reappearing only at degree 3; this is the electron, the family P20 calls character 0 and A287 sets at the anchored edge slot. The three coincide on the electron (Resolution A), which is why the electron is the normalization anchor; but this identity only relabels the special family and does not by itself fix which coefficient the other two draw.

3 The Structural Origin, and Why the Forcing Is Not Airtight

Remark 3.1 (The twist is one unit of ω between two genuinely different actions, and the missing bridge). *The twist’s origin is a handedness mismatch between two distinct actions of the one $\mathbb{Z}_3$.*

The layer cycle of P18 §3.5 is $\text{bulk} \rightarrow \text{boundary} \rightarrow \text{fiber} \rightarrow \text{bulk}$, that is degrees $3 \rightarrow 2 \rightarrow 1 \rightarrow 3$, a step that decreases degree. The graded action on monomials is $x^d \mapsto \omega^d x^d$, under which the character increases with degree. One layer-cycle step changes the graded character by a uniform $-1 \pmod{3}$: the two run in opposite directions on the degree ladder. The family index follows the layer cycle, since the family character is the T_{cycle} eigenvalue exponent; the coefficient-drawing follows the degree grading. Matching the two therefore costs exactly one unit of ω , a uniform twist by the ω character (Resolution C). The sign is not free: A304 forces the layer cycle forward, and the grading inherits $z \mapsto \omega z$ from the same forward rotation, so the residual is the genuine handedness gap, a fixed -1 , not a choosable ± 1 .

This is as far as forcing reaches, and it is honest to stop there. The reconciling convention is unique and its structural origin is named. What the corpus does not supply is the bridge that fixes which of the two actions the family index follows against the coefficient it draws. P18 and P20 define the family character as the T_{cycle} eigenvalue exponent, with no reference to ρ 's degree; A287 assigns by layer, with no reference to the eigenvalue. The reconciler is the value that bridge must take, namely that a family's T_{cycle} eigenvalue index k equals the ground-relative degree of the coefficient it draws, $k = (\text{degree} - 1) \pmod{3}$ with the edge as ground; but the bridge itself is never stated. Re-reading the existing corpus cannot close the gap, since under the forced generator neither the literal grading nor the Dirichlet shift reconciles and only the unstated edge-anchored rule does. The verdict is therefore PARTIAL, an open reindexing with a named, unique structural reconciler: the twist is not a free convention, but its vanishing is not forced by anything the corpus currently states.

The pin the corpus must add. One statement upgrades PARTIAL to HIT: a charged-lepton family's T_{cycle} eigenvalue index k equals the ground-relative degree of the ρ -coefficient it draws, $k = (\text{degree} - 1) \pmod{3}$, with the edge layer at degree one taken as the ground. This identifies the eigenvalue labelling of P20 with the layer labelling of A287, makes the forward generator's grading read off the ground-relative rather than the literal degree, and turns the uniform $+1$ twist into an identity. It is self-consistent and unique under A304's forced orientation; it is the single missing bridge.

Status. OI-287-1 remains open in its corrected form, and its standing residual is now precisely located. The mechanism is a $\mathbb{Z}_3$ character selection rule (A323), the coefficient set is the density's own $\mathbb{Z}_3$ character content (A324), and the family-to-coefficient labelling reconciles under a unique convention, the edge-anchored ground-relative degree, whose structural origin is the opposite handedness of the layer cycle and the degree grading, one unit of ω . The remaining open piece is one unstated corpus bridge, the equality of a family's eigenvalue index with the ground-relative degree it draws. The overlap-diagonal of A299 through A320 stays withdrawn; no wavefunction or overlap is formed. No published number changes; the A287 lepton law and its 10^{-4} accuracy are untouched.

Verifier. `verify_P325.py`: recomputes $c_n = (2, 3, 16)$ and $\rho(0) = 0$; the P20 and A287 labellings and the uniform $+1$ twist between them; the trivial-representation-is-anchor identity (Resolution A) and that it only relabels; the ground-relative index and that it, not the literal grading, maps the A287 degrees onto the P20 characters (Resolution B); the complete convention scan establishing the unique reconciler $\chi(d) = (d - 1) \pmod{3}$ under the A304-forced generator, with the literal and Dirichlet conventions failing; the handedness mismatch between the layer cycle and the degree grading as the structural origin, one unit of ω (Resolution C); the PARTIAL verdict; and the exact pin needed. Twenty-three checks, all PASS.