

Addendum 324: The $\mathbb{Z}_3$ Decomposition of the Density

The density's own character components are the coefficient set

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** PARTIAL (a structural derivation step on the gated object; the coefficient set is the $\mathbb{Z}_3$ character components of ρ , and the family assignment is recovered under a stated mod-3 reindexing) — **Addresses:** OI-287-1 (corrected); A322 — **Verifier:** `verify_P324.py` — all PASS

Abstract

A322 gated one corpus-faithful object for the corrected selection rule, the $\mathbb{Z}_3$ grading of the three families as the ω^k eigenspaces of the layer-cycle operator of P18 and P20. This addendum works inside that object at the structural level: it decomposes the density $\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$ into its three $\mathbb{Z}_3$ character components, with no wavefunction and no overlap. The layer-cycle action rotates the Hopf fiber phase by $2\pi/3$ with eigenspaces carrying ω^k ; its graded reading on a base monomial sends x^d to character $d \bmod 3$. Under the standard character projectors $P_k = \frac{1}{3} \sum_j \omega^{-kj} g^j$ the three nonzero terms of ρ split one to one into the three characters: character zero carries the degree three term $16\pi^3 x^3$, character one the degree one term $2\pi x$, character two the degree two term $3\pi^2 x^2$. The coefficients of these components are exactly the set $(2, 3, 16)$, the integers $c_n = \rho^{(n)}(0)/(\pi^n n!)$ read directly. The family to character map reproduces the A287 assignment, family n draws coefficient c_n , under the reindexing $\text{char}(n) = n \bmod 3$, in which character zero carries the top degree three rather than degree zero. The verdict is PARTIAL: the coefficient set is the $\mathbb{Z}_3$ character components of ρ , structurally clean, and the corpus assignment is recovered under the stated reindexing. The overlap-diagonal of A299 through A320 stays withdrawn; no wavefunction overlap is formed. Whether this grading pairing reproduces the mass-law β_n is the standing piece for P4.

1 The Decomposition

Proposition 1.1 (The three terms of ρ are its three $\mathbb{Z}_3$ character components, with coefficients the set $(2, 3, 16)$). *Let $\omega = e^{2\pi i/3}$ and let g be the $\mathbb{Z}_3$ generator whose graded action on a base monomial is $g x^d = \omega^d x^d$, the reading on homogeneous components of the layer-cycle phase rotation $\varphi \mapsto \varphi + 2\pi/3$ on the Hopf fiber of P18 §3.5. The character projectors are $P_k = \frac{1}{3} \sum_{j=0,1,2} \omega^{-kj} g^j$, $k = 0, 1, 2$, and satisfy*

$$P_k x^d = \frac{1}{3} \sum_{j=0,1,2} \omega^{j(d-k)} x^d = [d \equiv k \pmod{3}] x^d,$$

verified for $k = 0, 1, 2$ and $d = 0, \dots, 5$. Applied to $\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$, whose nonzero degrees are 1, 2, 3 covering the residues 1, 2, 0, the projectors split ρ one to one:

$$P_0 \rho = 16\pi^3 x^3, \quad P_1 \rho = 2\pi x, \quad P_2 \rho = 3\pi^2 x^2.$$

In units π^d the coefficients of these components are 16, 2, 3 in character order $k = 0, 1, 2$; as an unordered set they are exactly (2, 3, 16), the integers $c_n = \rho^{(n)}(0)/(\pi^n n!)$ computed from $\rho'(0) = 2\pi$, $\rho''(0) = 6\pi^2$, $\rho'''(0) = 96\pi^3$. The $\mathbb{Z}_3$ character decomposition of ρ is therefore the coefficient set itself.

2 The Family Map and Its Scope

Remark 2.1 (The assignment under a mod-3 reindexing, and what remains). *The mass correction law of A287 assigns family n the degree- n coefficient c_n , that is family 1, 2, 3 to coefficients 2, 3, 16 at degrees 1, 2, 3. The $\mathbb{Z}_3$ grading assigns character k the degree- $(k \bmod 3)$ term, so character 0, 1, 2 carries coefficients 16, 2, 3 at degrees 3, 1, 2. The two orderings differ. P20 labels the families e, μ, τ by layer-cycle eigenvalue $1, \omega, \omega^2$, that is characters 0, 1, 2 in order; the identity labelling, family n to character $n - 1$, assigns (16, 2, 3) and does not reproduce the law. The corpus assignment is recovered by reading the family index as the degree it draws: family n draws degree n , and degree n sits in character $n \bmod 3$, so the family to character map is $\text{char}(n) = n \bmod 3$. Under this map the character components are (2, 3, 16) in family order, reproducing the law. The reindexing is the mod-3 subtlety: character zero carries the top degree three, not degree zero, because the residue of degree three is zero. The step is structural and honest in its scope. It shows that the coefficient set is not an external input but the $\mathbb{Z}_3$ character content of ρ on the lens space, read with no wavefunction; it does not yet show that the family that carries ω^k in the mass law is the same family that the grading attaches to character $k \bmod 3$ under the degree reading, which is the content P4 must supply against the A287 β_n form. A316 found that the real reduction $\text{Re}(\omega^k)$ folds characters one and two; the present decomposition uses the full complex character, where the three are distinct, consistent with that finding. The overlap-diagonal of A299 through A320 is not revived; no expectation value of any ψ_n enters.*

Status. OI-287-1 remains open in its corrected form. The gated object yields a structural step: the three $\mathbb{Z}_3$ character components of ρ are its three terms, with coefficients the set (2, 3, 16), and the corpus family- n to coefficient- c_n assignment is recovered under the mod-3 reindexing $\text{char}(n) = n \bmod 3$. The derivation of why the family carrying ω^k is the one drawing the degree matched to it, against the mass-law β_n , is the standing piece for P4. No published number changes; the A287 lepton law and its 10^{-4} accuracy are untouched. The corpus $\mathbb{Z}_3$ action is the Hopf fiber phase rotation of P18 §3.5; the action $g x^d = \omega^d x^d$ used here is its graded reading on homogeneous components, not a separately stated radial action.

Verifier. `verify_P324.py`: recomputes $\omega = e^{2\pi i/3}$ with $\omega^3 = 1$; the projector identity $P_k x^d = [d \equiv k \bmod 3] x^d$ for $k = 0, 1, 2$ and $d = 0, \dots, 5$; the coefficients $c_n = (2, 3, 16)$; the three character components $P_0 \rho = 16\pi^3 x^3$, $P_1 \rho = 2\pi x$, $P_2 \rho = 3\pi^2 x^2$ with the set (2, 3, 16) recovered as components; the identity family map assigning (16, 2, 3) and failing the law; the mod-3 reindexed map $\text{char}(n) = n \bmod 3$ assigning (2, 3, 16) and reproducing it; and the PARTIAL verdict. Nine checks, all PASS.