

Addendum 319: The Boundary S^3 Problem Directly

The pure-boundary harmonic and lens-space reductions overshoot the magnitude and do not reach the gap the radial reduction missed

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (null; the pure-boundary S^3 reduction breaks the family degeneracy but overshoots the magnitude, does not reach the gap the radial reduction could not, and does not improve on the radial-best fit) — **Addresses:** OI-287-1 — **Verifier:** `verify_P319.py` — all PASS

Abstract

The cumulative arc through Addendum 318 established that the magnitude residual of the charged-lepton selection rule is not in the radial reduction by any channel. Across radial channels the μ entry runs pure-radial 1.222, r^3 measure 1.337, fiber m^2 energy 2.3, and Dirac first-order 3.14, while the target $\mu = 1.5$ sits in a gap, $[1.34, 2.0]$, reached by none: the channels that carry the cube-root phase overshoot it and the channels that do not undershoot. This addendum leaves the radial bound-state picture and builds the boundary S^3 problem directly. The objects are the $SO(4)$ harmonics, degree L with Laplace-Beltrami eigenvalue $L(L+2)$ and multiplicity $(L+1)^2$, built from the $\mathfrak{so}(4) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$ Casimir as in Addendum 314, and the $\mathbb{Z}_3 = S^3/\mathbb{Z}_3$ lens-space characters, the ω^k eigenspaces of the $2\pi/3$ Hopf rotation. Two formulations are declared in advance: B -harm, with families the lowest harmonic degrees $L = 0, 1, 2$, and B -lens, the canonical reading per Paper 20, with families the three $\mathbb{Z}_3$ character classes on $L(3, 1)$. The boundary degree- n coupling is the harmonic overlap, the moment $c_n \pi^n$ carried by the family- n sector's harmonic multiplicity, normalized by its $SO(4)$ Casimir spectral weight, the boundary analog of the radial $\langle \psi_n | c_n \pi^n r^n | \psi_n \rangle / \lambda_n$. Both formulations break the family degeneracy and give three distinct entries, but both overshoot the magnitude: B -lens returns $\mu = 3.86$ and B -harm $\mu = 6.28$, both far above the target and above the radial fiber overshoots. The boundary problem does not reach the gap, overshooting straight past it, and does not improve on the radial-best distance 0.399. The verdict is null: the magnitude is in neither the radial nor the pure-boundary reduction.

1 The Probe

Proposition 1.1 (The pure-boundary reduction overshoots the magnitude and does not reach the gap). *On the boundary $S^3 = \partial B^4$ the natural objects are the $SO(4)$ scalar harmonics, degree L carrying the Laplace-Beltrami eigenvalue $L(L+2)$ with multiplicity $(L+1)^2$, and the lens-space $L(3, 1) = S^3/\mathbb{Z}_3$ character classes, the eigenspaces of the layer-cycle under which the Hopf fiber rotates by $2\pi/3$. The harmonic data is built from the $\mathfrak{so}(4) = \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R$ Casimir on the symmetric (j, j) irreps, $4j(j+1) = L(L+2)$ with $L = 2j$, the same construction Addendum 314 used, giving $L(L+2) = 0, 3, 8$ and multiplicity $1, 4, 9$ for $L = 0, 1, 2$. The boundary degree- n density coupling is declared as the harmonic overlap, the moment $c_n \pi^n$ carried by the family- n sector's harmonic multiplicity and normalized by that sector's spectral weight, the boundary analog*

of the radial matrix element over its eigenvalue, with the constant harmonic's zero Laplacian energy handled by the multiplicity floor $(L+1)^2 = 1$. Two formulations are tested, both declared before any number is computed. In *B-harm* the families are the lowest harmonic degrees $L = 0, 1, 2$, and the normalized diagonal is $(1, 6.283, 88.83)$ with $\mu = 6.283$ and $\tau = 88.83$, at distance 80.97 to the target $(1, 1.5, 8)$. In *B-lens*, the canonical reading in which the three families are the $\mathbb{Z}_3$ character classes of the lens space, the normalized diagonal is $(1, 3.856, 64.60)$ with $\mu = 3.856$ and $\tau = 64.60$, at distance 56.65. Each formulation produces three distinct entries with no degenerate collapse, since the three character classes are genuinely populated and distinct, the complex ω^k structure native on the lens space. But each overshoots the magnitude: both μ values lie above the target 1.5 and above the radial fiber overshoots of 2.3 to 3.14, and neither restores a valid column-standardized peak against the flat control. Neither reaches the gap $[1.34, 2.0]$ the radial reduction could not enter, overshooting straight past it, and neither improves on the radial-best distance 0.399 at $\mu = 1.222$.

Remark 1.2 (The verdict is null; the boundary alone is not the arena). *The construction is the boundary problem the cumulative radial arc pointed to, built without fitting. The boundary objects are the corpus ones, the $SO(4)$ harmonics and the lens-space characters, and the harmonic data is the genuine $\mathfrak{so}(4)$ Casimir spectrum machine-built as in Addendum 314, not assumed. The canonical family definition is the Paper 20 reading, the three families as the $\mathbb{Z}_3$ character classes carrying the ω^k phase. The first result is structural: both formulations break the degeneracy that defeated the real radial reduction in Addendum 316, since the three character classes are distinct on the lens space, where the real layer-cycle shift folded the second and third families together. The second result is the magnitude, and here the boundary reduction does not close. Both formulations overshoot, *B-lens* to $\mu = 3.86$ and *B-harm* to $\mu = 6.28$, with τ far above the target 8, and neither restores a peak. The pattern that held across the radial channels holds again on the boundary, in sharper form: the structure that carries the cube-root phase overshoots the magnitude, now past even the radial overshoots. The target $\mu = 1.5$ sits in the gap $[1.34, 2.0]$, and the boundary problem does not enter it. The reading is that the magnitude is in neither the radial reduction nor the pure-boundary reduction. The boundary carries the family distinction faithfully, as the radial fiber and Dirac channels did, but the magnitude is not a property of the boundary spectral weights any more than it was of the radial ones. What remains untested is the full bulk-and-boundary coupled problem, in which the radial bulk states and the boundary harmonics enter together rather than either reduction alone. The magnitude piece of OI-287-1 does not close here.*

Status. OI-287-1 remains open. A319 leaves the radial reduction and tests the boundary S^3 problem directly, in two formulations declared in advance: *B-harm*, families the lowest $SO(4)$ harmonic degrees $L = 0, 1, 2$, and *B-lens*, the canonical reading with families the three $\mathbb{Z}_3$ character classes on the lens space $L(3, 1)$. Both break the family degeneracy and give three distinct entries, the character classes genuinely distinct on the lens space. But both overshoot the magnitude: *B-lens* returns $\mu = 3.86$ at distance 56.65 and *B-harm* $\mu = 6.28$ at distance 80.97, both above the radial fiber overshoots, and neither restores a peak. The boundary problem does not reach the gap $[1.34, 2.0]$ the radial reduction could not enter, and does not improve on the radial-best distance 0.399 at $\mu = 1.222$. The pure-boundary reduction therefore supplies the family distinction but not the magnitude, the same shape of result the radial fiber and Dirac channels gave. No published number changes. The magnitude is shown to be in neither the radial nor the pure-boundary reduction; the remaining arena is the full bulk-and-boundary coupled problem, in which the radial states and the boundary harmonics enter together.

Verifier. `verify_P319.py`: rebuilds the $SO(4)$ harmonic spectrum from the $\mathfrak{so}(4)$ Casimir and confirms it is the genuine boundary data, $L(L+2) = 0, 3, 8$ with multiplicity $(L+1)^2 = 1, 4, 9$ for $L = 0, 1, 2$ as in Addendum 314; confirms that each formulation gives three distinct families with all three $\mathbb{Z}_3$ classes populated; recomputes the B -harm diagonal $(1, 6.28, 88.8)$ at distance 80.97 and the B -lens diagonal $(1, 3.86, 64.6)$ at distance 56.65, with no valid peak in either; confirms that the boundary problem does not reach the gap $[1.34, 2.0]$, both μ above 2.0; and confirms the null verdict's defining condition, that the canonical map does not hit, the boundary does not reach the gap, and the boundary does not beat the radial-best distance 0.399, with no NaN. Eight checks, all PASS.