

Addendum 318: The First-Order Radial Dirac Reduction

The phase carried through the Dirac off-diagonal coupling sharpens the overshoot rather than softening it

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (null; the first-order Dirac reduction does not soften the A317 magnitude overshoot and does not improve on the scalar-radial closest) —
Addresses: OI-287-1 — **Verifier:** `verify_P318.py` — all PASS

Abstract

A317 realized the three charged-lepton families as the fiber-momentum sectors $m \equiv k \pmod{3}$ that carry the cube-root phase ω^m , broke the A316 degeneracy, and found that both corpus-faithful fiber forms enter the phase as an additive energy m^2 and overshoot, returning $\mu = 2.29$ to 2.43 with no peak. The entry A317 left untested was the phase entering through the first-order Dirac structure rather than as additive energy: the full B^4 spinor problem of P18 S2.2 and S3.1, where the master operator acts on sections of a spinor bundle, rather than the scalar radial reduction used from A296 through A317. This addendum builds that object. The four-dimensional radial Dirac operator couples an upper and a lower radial function (f, g) through a first-order system with the off-diagonal terms $-d/dr - (\kappa + c)/r$ and its adjoint, where κ is the boundary-Dirac eigenvalue carrying the family label and $c = 3/2$ is the four-dimensional constant fixed in A312 and A314. The canonical map sends family n to the n -th boundary-Dirac sector $\kappa_n = (n - 1) + 3/2$, the $\pm(j + 3/2)$ tower of A314 that is the spinor realization of the ω^k sectors. The density overlap is formed as the spinor bilinear $f^2 + g^2$ in the r^3 measure, divided by the eigenvalue. The construction is stable and the boundary-Dirac ordering is recovered: the free-field lowest modes are monotone in κ . The three families are genuinely distinct, so the A316 degeneracy is broken in this reduction as well. But the first-order coupling does not soften the magnitude. The canonical map returns $\mu = 3.14$ and $\tau = 38.4$, above A317's additive overshoot rather than below it, at distance 30.5 to the target, with no peak. The verdict is null on the magnitude piece: the first-order Dirac structure carries the phase into the matrix element more strongly than the additive energy, not more gently, and it does not improve on the scalar-radial closest at distance 0.399.

1 The Probe

Proposition 1.1 (The radial Dirac reduction is stable and family-distinct, and its canonical map overshoots above A317). *On the geometry of P18 S2.1, with metric $ds^2 = dr^2 + r^2 d\Omega_3^2$ and the master operator on sections of a spinor bundle (P18 S2.2), the four-dimensional radial Dirac operator acts on the two-component spinor $\Psi = (f, g)^T$, self-adjoint in $r^3 dr$, through the first-order system with diagonal mass $m(r) = \sqrt{V_{\text{eff}}(r)}$ and off-diagonal coupling $-d/dr - (\kappa + c)/r$ together with its adjoint $d/dr - (\kappa - c)/r$, where $V_{\text{eff}}(r) = \rho'(r)^2/(2\Omega^2) + E_{\text{self}}r^2(1 - r)^2$ is the potential of the scalar thread, κ is the boundary-Dirac eigenvalue, and $c = 3/2$ is the four-dimensional family constant of A312 and A314 equal to the Lichnerowicz constant $R/4$. The canonical map sends family n to*

the n -th boundary-Dirac sector $\kappa_n = (n-1) + 3/2 = 3/2, 5/2, 7/2$, the positive $\pm(j+3/2)$ tower of A314, and the diagonal entry n is the degree- n density overlap of that sector's lowest bound state, formed as the spinor bilinear $f^2 + g^2$ in $r^3 dr$ and divided by the eigenvalue. The discretization on a staggered grid is stable, with no NaN, and the free-field lowest modes are monotone in κ , $5.76 < 6.99 < 8.18$, recovering the boundary-Dirac tower ordering of A314 in the four-ball radial spectrum. The three sectors give three distinct Dirac eigenvalues, 6.48, 7.74, 8.94, and three distinct diagonal entries with no collapse, so the A316 degeneracy is broken here as it was in A317. But the canonical map returns the diagonal (1, 3.143, 38.430) with $\mu = 3.143$ and $\tau = 38.430$, at distance 30.47 to the target (1, 1.5, 8). This is above A317's additive overshoot of $\mu = 2.29$ to 2.43, not below it. The map lands no entry within twenty percent of the target and restores no valid peak, the column-standardized argmax of column n equal to n for all three columns while the sine-box control formed in the same measure fails the same test. It does not improve on the scalar-radial closest of A316 at distance 0.399 and $\mu = 1.222$.

Remark 1.2 (The first-order coupling sharpens the overshoot; what remains). *The construction is the entry A317 named, built without fitting. The family definition is the corpus one, the families as the boundary-Dirac sectors that carry the ω^k labels, and the phase enters through the first-order off-diagonal coupling $(\kappa \pm c)/r$ rather than the additive m^2 of A317. The prediction under test was that this first-order channel would carry the phase into the matrix element gently, through the relative weight of the upper and lower components, and move μ down from A317's overshoot toward 1.5. It does not. The off-diagonal $(\kappa + c)/r$ term grows with κ , so the higher families acquire a larger lower component and a more sharply weighted density, and the degree- n overlap rises faster than the additive energy raised it. The result is $\mu = 3.14$ where A317 gave 2.29 to 2.43 and the scalar radial reduction gave 1.222. The first-order structure is the gentler-coupling hope of A317 made explicit, and the explicit form refutes it: carrying the phase through the Dirac off-diagonal coupling is stronger than carrying it through the additive energy, not weaker. The two structural facts that survive are the same two A317 reported. The reduction is stable and corpus-faithful, the free-field modes sit in the A314 tower ordering, and the three families are genuinely distinct, so the A316 degeneracy is broken in the spinor reduction as well as the scalar one. But the magnitude is not supplied. The factor that lifts the second family to exactly 1.5 and sharpens the diagonal to a peak is now excluded from a further channel: it is neither a measure choice, nor an angular coupling strength, nor a real family-to-mode selection, nor the fiber-momentum eigenvalue of the complex realization, nor the first-order Dirac off-diagonal coupling of the spinor reduction. Both ways of carrying the ω^k phase into the radial problem, the additive energy and the first-order coupling, overshoot. The residual is not in how the phase enters the radial matrix element at all. What remains untested is a reduction that is not radial: the boundary S^3 problem directly, or a coupling between the degree- n density and the spectrum that does not pass through a radial bound state. The magnitude piece of OI-287-1 does not close here.*

Status. OI-287-1 remains open. A318 builds the first-order B^4 radial Dirac system that A317 named as the untested entry, with the families as the boundary-Dirac sectors $\kappa_n = (n-1) + 3/2$ that carry the ω^k labels and the phase entering through the off-diagonal coupling rather than the additive energy. The reduction is stable, recovers the A314 boundary-Dirac tower ordering in its free-field modes, and breaks the A316 degeneracy with three distinct families. But the canonical map overshoots above A317, returning $\mu = 3.14$ and $\tau = 38.4$ at distance 30.5, where A317's additive forms gave $\mu = 2.29$ to 2.43 and the scalar radial reduction gave $\mu = 1.222$ at distance 0.399. The first-order Dirac coupling carries the ω^k phase more strongly than the additive energy, not more gently, so it does not soften the magnitude. No published number changes. The residual

is now excluded from how the phase enters the radial matrix element in any of its tested channels; what remains is a non-radial reduction of the master operator.

Verifier. `verify_P318.py`: recomputes the radial Dirac system from scratch and confirms that it solves with no NaN on the staggered grid, that the three sectors $\kappa = 3/2, 5/2, 7/2$ give three distinct Dirac eigenvalues and three distinct diagonal entries with no collapse, that the free-field lowest modes are monotone in κ recovering the A314 boundary tower ordering, that the canonical map returns the diagonal $(1, 3.143, 38.430)$ with $\mu = 3.143$ and $\tau = 38.430$ overshooting the target, that no valid peak is restored and no entry lands within twenty percent of $(1, 1.5, 8)$, that the first-order coupling is not gentler than A317's m^2 overshoot since $\mu = 3.14$ exceeds 2.29 to 2.43, and that the null verdict's defining condition holds, the reduction improving neither on the scalar-radial closest at distance 0.399 nor on A317. Nine checks, all PASS.