

Addendum 317: The Complex Hopf-Fibered Realization

The genuine cube-root phase breaks the degeneracy but does not supply the magnitude

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (null; the complex realization breaks the A316 family degeneracy but does not reproduce the magnitude target with a peak, and does not improve on the real-reduction closest) — **Addresses:** OI-287-1 — **Verifier:** verify_P317.py — all PASS

Abstract

A316 closed the family-to-mode route in the real radial reduction of the charged-lepton selection rule and located the blocker in the complex structure of the family definition. The three families are the three cube-root eigenspaces ω^k of the layer-cycle, $\omega = e^{2\pi i/3}$, but the real radial reduction folds ω and ω^2 together: the real layer-cycle shift $\text{Re}(\omega^k) = (1, -\frac{1}{2}, -\frac{1}{2})$ takes equal values on $k = 1$ and $k = 2$, so families two and three collapse to one radial state. This addendum moves to a complex realization. On the Hopf fibration $S^1 \hookrightarrow S^3 \rightarrow S^2$ the layer-cycle acts as a rotation $\psi \mapsto \psi + 2\pi/3$ of the fiber coordinate, so on the fiber-momentum sector $e^{im\psi}$ it multiplies by $e^{im2\pi/3} = \omega^m$, and the ω^k eigenspace is the set $\{m \equiv k \pmod{3}\}$ with lowest representative $m = k$. The fiber momentum enters the radial operator through the fiber Laplacian, whose eigenvalue is m^2 , and $m = 0, 1, 2$ give three distinct values where $\text{Re}(\omega^k)$ gave only two. Two corpus-faithful forms of the fiber term are tested, an additive m^2 and a cone-weighted m^2/r^2 , with the canonical map sending family k to the ground radial state of the $m = k$ sector. The complex realization breaks the A316 degeneracy: $m = 0, 1, 2$ are three genuinely distinct operators and produce three distinct diagonal entries. But the canonical map overshoots, returning $\mu = 2.43$ for the additive form and $\mu = 2.29$ for the cone form, both far above the target 1.5, with no peak, and neither improves on the real-reduction closest at distance 0.399 and $\mu = 1.222$. The verdict is null on the magnitude piece: the genuine cube-root phase supplies the broken degeneracy, a real structural result, but not the magnitude, which lies elsewhere.

1 The Probe

Proposition 1.1 (The complex realization breaks the degeneracy; the canonical map overshoots the magnitude). *On the forced arena geometry of A311, the four-dimensional radial operator $-d^2/dr^2 - (3/r)d/dr + V_{\text{eff}}(r)$, self-adjoint in $r^3 dr$ with both eigenstates and the degree- k density overlap formed in $r^3 dr$, gains the fiber-momentum term of the Hopf fibration. The layer-cycle rotates the fiber coordinate by $2\pi/3$, so the ω^k eigenspace is the fiber-momentum sector $m \equiv k \pmod{3}$ with lowest representative $m = k$, and the fiber Laplacian contributes the eigenvalue m^2 . Two corpus-faithful forms are tested: the additive form $\hat{O}_m = -d^2/dr^2 - (3/r)d/dr + V_{\text{eff}}(r) + m^2$, the fiber as a circle of fixed circumference, and the cone form $\hat{O}_m = -d^2/dr^2 - (3/r)d/dr + m^2/r^2 + V_{\text{eff}}(r)$, the fiber scaling with the radius like the angular term $l(l+2)/r^2$. The canonical map sends family k to the ground radial state of the $m = k$ sector, $k = 0, 1, 2$, and the diagonal entry $n = k + 1$ is*

the degree- n density overlap of that state divided by its eigenvalue. The three sectors $m = 0, 1, 2$ are three distinct operators: the additive form gives ground eigenvalues 23.27, 24.27, 27.27 and the cone form 23.27, 29.31, 42.39, all distinct, and at $m = 0$ both forms reduce to the pure radial operator with ground eigenvalue 23.27. Each form returns three distinct diagonal entries, with no degenerate collapse, unlike the real reduction of A316. The additive form returns the diagonal (1, 2.427, 21.040) with $\mu = 2.427$ and $\tau = 21.040$, at distance 13.07 to the target; the cone form returns (1, 2.290, 20.726) with $\mu = 2.290$ and $\tau = 20.726$, at distance 12.75. Neither lands within twenty percent of (1, 1.5, 8) and neither restores a valid peak, the column-standardized argmax of column k equal to k for all three columns while the sine-box control formed in the same measure fails the same test. Neither improves on the real-reduction closest of A316, distance 0.399 at $\mu = 1.222$.

Remark 1.2 (The verdict is null; what the cube-root phase did and did not supply). The construction is the complex realization A316 pointed to, built without fitting. The family definition is the corpus one, the families as the ω^k eigenspaces of the layer-cycle, and the realization carries the cube-root phase genuinely: because the layer-cycle is a rotation of the Hopf fiber, the eigenspaces are the fiber-momentum sectors $m \equiv k \pmod{3}$, and the lowest representatives $m = 0, 1, 2$ enter the radial operator through the fiber-Laplacian eigenvalue m^2 . The first result is structural and clean. The three sectors are three distinct operators, $m^2 = 0, 1, 4$ in the additive form and m^2/r^2 in the cone form, and they give three distinct diagonal entries with no collapse. This is the quantity A316 could not produce: the real layer-cycle shift took two equal values on the second and third families, and the real radial reduction had no further structure to separate them; the complex fiber momentum separates them, since $1 \neq 4$ where $\text{Re}(\omega) = \text{Re}(\omega^2)$. The degeneracy that A316 named as the blocker is removed. The second result is the magnitude, and here the canonical map does not close. Both forms send μ well above the target rather than to it: the additive form to 2.43 and the cone form to 2.29, with τ above 20 where the target is 8, and neither restores a peak. The genuine cube-root phase, carried through the only corpus-faithful fiber forms, overshoots. It does not move μ toward 1.5 from the real-reduction value 1.222; it pushes it past, and the distance to the full target grows from 0.399 to roughly thirteen. A hit would have required the canonical map, family k to the $m = k$ sector, to land the target; it does not, and no non-canonical reassignment of m is invoked, so there is no fit to report and no derivation to claim. The reading is that the complex Hopf phase supplies the structure A316 was missing, three distinct families, but not the magnitude. The factor that lifts the second family to exactly 1.5 and sharpens the diagonal to a peak is not the fiber-momentum eigenvalue, in either of its corpus-faithful forms. It lies elsewhere: in how the fiber sector couples to the density overlap rather than in the additive or centrifugal energy it contributes, or in a different reduction of the master operator than the radial one this thread has used throughout. The magnitude piece of OI-287-1 does not close here.

Status. OI-287-1 remains open. A317 confirms the structural prediction of A316 and refutes the magnitude hope built on it. The complex Hopf-fibered realization, with the families as the fiber-momentum sectors $m \equiv k \pmod{3}$ that carry the cube-root phase ω^m , breaks the A316 degeneracy: $m = 0, 1, 2$ are three distinct operators and give three distinct diagonal entries, where the real layer-cycle shift gave only two. But the canonical map, family k to the ground state of the $m = k$ sector, overshoots the magnitude in both corpus-faithful fiber forms, returning $\mu = 2.43$ for the additive m^2 term and $\mu = 2.29$ for the cone m^2/r^2 term, with τ above 20 and no peak in either. Neither improves on the real-reduction closest of A316, which stands at distance 0.399 and $\mu = 1.222$. The genuine cube-root phase therefore supplies the broken degeneracy, a real result, but not the magnitude. No published number changes. The remaining quantity is the factor that lifts the second family to 1.5 and restores the diagonal peak, now shown to be neither a measure

choice, nor an angular coupling strength, nor a family-to-mode selection in the real reduction, nor the fiber-momentum eigenvalue of the complex realization; the residual lies in the coupling of the fiber sector to the overlap or in a reduction of the master operator other than the radial one.

Verifier. `verify_P317.py`: recomputes both fiber forms from scratch and confirms that $m = 0, 1, 2$ give three distinct operators in each form, that both forms reduce to the pure radial operator at $m = 0$ with ground eigenvalue 23.27, that the additive form returns $(1, 2.427, 21.040)$ at distance 13.07 and the cone form $(1, 2.290, 20.726)$ at distance 12.75, that neither lands within twenty percent of the target and neither restores a valid peak, that the A316 degeneracy is broken with three distinct entries and no collapse in both forms, and that the complex realization does not improve on the real-reduction closest at distance 0.399 and $\mu = 1.222$, so the null verdict's defining condition holds; and no NaN appears. Eight checks, all PASS.