

Addendum 316: The Family-to-Mode Selection Rule

No principled mode map reproduces the magnitude target with a peak

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (null; neither corpus-canonical family-to-mode map reproduces the target diagonal with a valid peak, and no declared map does) — **Addresses:** OI-287-1 — **Verifier:** verify_P316.py — all PASS

Abstract

A311 fixed the inner-product measure of the charged-lepton selection rule as the geometrically forced radial volume $r^3 dr$ of the B^4 arena, and A315 showed the residual to the target ratio $\mu = 1.5$ is not a one-parameter angular coupling strength: the magnitude motion lives in which eigenstate of the forced four-dimensional radial operator each family occupies, the family-to-mode selection rule. This addendum enumerates five principled family-to-mode maps, declared before any number is computed, and tests for each whether it reproduces the target diagonal (1, 1.5, 8) within twenty percent with a valid column-standardized peak. Two of the five are privileged by the corpus as the two principled family definitions: the node-count map, taking family n to the $(n-1)$ -th radial mode of the $l = 0$ operator, and the $\mathbb{Z}_3$ -character map, taking family n to the ω^k eigenspace of the layer-cycle operator, $k = n-1$. A hit requires one of these two canonical maps; a fit by a non-canonical map is reported as a fit. The finding is null. The node-count map returns the diagonal (1, 1.222, 8.286), the closest of all five maps and identical to the pure radial object of A311 and A313, but μ falls short of 1.5 and no peak is restored. The $\mathbb{Z}_3$ -character map, realized through the real layer-cycle shift $\text{Re}(\omega^k)\gamma$ the radial reduction admits, is rank-degenerate: the shifts $\gamma(1, -\frac{1}{2}, -\frac{1}{2})$ make the $k = 1$ and $k = 2$ operators identical, so families two and three collapse to one radial state and the map cannot produce three distinct diagonal entries. The non-canonical maps either overshoot, with the angular-sector map at $\mu = 2.009$ and the shifted-sector map at $\mu = 2.326$, or reduce to the node-count map, as the self-consistent diagonal map does, selecting modes 0, 1, 2 and giving no peak even when it selects for one. No declared map both lands within twenty percent of the target and restores a valid peak. The verdict is null: the family-to-mode rule that lands the target with a peak is not among the principled maps, and neither corpus-canonical map closes the magnitude piece.

1 The Probe

Proposition 1.1 (Neither corpus-canonical mode map hits the target with a peak). *On the forced arena geometry of A311, the four-dimensional radial operator $\hat{O}_l = -d^2/dr^2 - (3/r)d/dr + l(l+2)/r^2 + V_{\text{eff}}(r)$, self-adjoint in $r^3 dr$ with both eigenstates and the degree- k density overlap formed in $r^3 dr$, consider five family-to-mode maps. The node-count map (S -radial) takes family n to the $(n-1)$ -th radial mode of $\hat{O}_0$; the angular-sector map (S -lsector) takes family n to the ground mode of $\hat{O}_{n-1}$; the shifted-sector map (S -lground) takes family n to the ground mode of $\hat{O}_n$; the $\mathbb{Z}_3$ -character map (S -Z3) takes family n to the ground mode of $\hat{O}_0 + \gamma \cos(2\pi(n-1)/3)$, with $\gamma = 3/4$ the layer-cycle coefficient, the real footprint of the ω^k phase on the radial operator; the self-consistent map (S -diag) takes family n to the low mode of $\hat{O}_0$ whose degree- n overlap is maximal. Each diagonal*

entry n is the degree- n density overlap of the selected state divided by its eigenvalue, and a valid peak requires the column-standardized argmax of column k to equal k for all three columns while the sine-box control formed in the same measure fails the same test. Computed on the standard grid, the node-count map returns the diagonal $(1, 1.222, 8.286)$ with $\mu = 1.222$ and $\tau = 8.286$, node counts $0, 1, 2$, and distance 0.399 to the target, the closest of the five and identical to the A313 pure radial object; it restores no valid peak and μ is short of 1.5 . The $\mathbb{Z}_3$ -character map returns $\mu = 2.656$ and is rank-degenerate: the shifts $\gamma(1, -\frac{1}{2}, -\frac{1}{2})$ make the $k = 1$ and $k = 2$ operators identical, so families two and three share an eigenvalue and an entire overlap row, and the map yields only two distinct radial states. The angular-sector map gives $\mu = 2.009$, the shifted-sector map $\mu = 2.326$, and the self-consistent map reduces to the node-count map at $\mu = 1.222$ with no peak. No declared map both lands within twenty percent of $(1, 1.5, 8)$ and restores a valid peak; in particular neither of the two corpus-canonical maps does.

Remark 1.2 (The verdict is null; what remains). *The two maps privileged by the corpus are the two principled definitions of a fermion family. The node-count reading, that the three families are the three lowest radial modes distinguished by node count, is the one carried through the selection-rule thread; the $\mathbb{Z}_3$ -character reading, that the families are the three eigenspaces of the layer-cycle operator, is the one stated in the family-origin proposition of the embedding paper. Both were declared before computing, so a hit by either would have been a genuine derivation. Neither hits. The node-count map is the closest object in the entire arc but stops at the same $\mu = 1.222$ the pure radial operator gives, with the diagonal peak still unrecovered. The character map, in the only form the real radial reduction admits, is degenerate by construction: the real part of the cube-root phase takes two equal values, and the radial operator has no further structure to separate them, so the map collapses two of the three families. This is itself an honest reading of the gap. The $\mathbb{Z}_3$ phase is a complex label; the radial reduction is real, and the part of the family structure that distinguishes the second and third families is exactly the part the reduction cannot carry. Among the non-canonical maps, the angular and shifted-sector maps overshoot the target and the self-consistent map, which selects modes to maximize the diagonal, recovers only the node-count map and still produces no peak, which says the peak is not available to any selection among the low radial modes of a single operator. The closest fit on the full target is unchanged from A311 and A313. The verdict is null. The family-to-mode rule that both lands $(1, 1.5, 8)$ and restores the peak is not among the principled maps tested here, and the corpus-canonical maps, the node-count and the $\mathbb{Z}_3$ -character definitions, do not close the magnitude piece. What remains is a family-to-mode rule that carries the complex $\mathbb{Z}_3$ phase, or a structure beyond the real radial reduction, capable of distinguishing all three families and producing a diagonal peak.*

Status. OI-287-1 remains open. A316 closes the family-to-mode route as posed: among five pre-declared principled maps, neither corpus-canonical map, the node-count map nor the $\mathbb{Z}_3$ -character map, reproduces the target diagonal $(1, 1.5, 8)$ within twenty percent with a valid peak, and no non-canonical map does either. The node-count map is the closest, returning $(1, 1.222, 8.286)$ at distance 0.399 , the same pure radial object as A311 and A313, short of $\mu = 1.5$ with no peak. The $\mathbb{Z}_3$ -character map is rank-degenerate in the real radial reduction, collapsing two of the three families, because the real part of the cube-root phase takes two equal values. The closest fit is still the pure radial four-dimensional operator, and the diagonal peak remains unrecovered. No published number changes. The remaining quantity is a family-to-mode rule that carries the complex $\mathbb{Z}_3$ phase or reaches beyond the real radial reduction, together with the unrecovered peak.

Verifier. `verify_P316.py`: recomputes all five maps from scratch and confirms the node-count map reproduces the pure radial diagonal $(1, 1.222, 8.286)$ at distance 0.399, the closest map, with μ short of 1.5 and no valid peak; the angular-sector map overshoots at $\mu = 2.009$; the shifted-sector and self-consistent maps do not hit, the latter reducing to the node-count map; the $\mathbb{Z}_3$ -character map is rank-degenerate with families two and three collapsed; no canonical map and no non-canonical map hits, so the null verdict's defining condition holds, that no declared map both lands within twenty percent and restores a valid peak; and no NaN appears. Eight checks, all PASS.