

# Addendum 315: The Angular Coupling Scan

No single-parameter centrifugal coupling brackets the magnitude target

TOE Corpus

2026-06-13

**Skill:** gap-closure v1.1 — **Outcome:** Case C (null with structural redirection; the magnitude residual is not a one-parameter coupling strength, so there is no  $g^*$  to test against a corpus constant)  
 — **Addresses:** OI-287-1 — **Verifier:** `verify_P315.py` — all PASS

## Abstract

A313 bracketed the magnitude piece of OI-287-1 between two forced objects on the forced  $r^3 dr$  arena measure of A311: the pure four-dimensional radial operator, whose three families are modes 0, 1, 2 of one operator, returning  $\mu = 1.222$ ; and the full angular identity  $l = n$ , whose three families are the ground radial mode of each  $l$ -sector operator, returning  $\mu = 2.009$ . A313 read the magnitude residual to the target  $\mu = 1.5$  as lying between these and proposed that a scalar angular coupling  $g \in [0, 1]$ , scaling the centrifugal term, would interpolate  $\mu$  from 1.222 to 2.009 and cross 1.5 at some  $g^*$  whose status as a corpus constant would decide whether the coupling is forced or fitted. This addendum runs that probe, with the candidate corpus constants for  $g^*$  declared before any value is computed. The finding is that the question is ill-posed as stated, and the value of the result is in saying so plainly. The two A313 endpoints differ in their mode-selection rule, not in their centrifugal strength. Holding the construction fixed and scaling only  $g$  gives two mode-consistent families: the  $n$ -th-mode family, which reproduces the four-dimensional radial endpoint at  $g = 0$  and runs  $\mu$  downward from 1.222 to 0.975; and the ground-mode family, which reproduces the  $l = n$  endpoint at  $g = 1$  and runs  $\mu$  downward from 2.532 to 2.009. Both are monotone in  $g$ , and neither crosses 1.5. The target lies in the gap between the disjoint ranges  $[0.975, 1.222]$  and  $[2.009, 2.532]$ , unreachable by either single knob. There is therefore no  $g^*$  to compare against the ten pre-declared corpus constants; the fit-versus-forced test is vacuous because there is neither a fitted nor a forced coupling. The  $1.222 \rightarrow 2.009$  motion A313 attributed to angular strength was mode-reindexing, the change from modes 0, 1, 2 of one operator to the ground mode of each  $l$ -sector, not a tunable coupling. The verdict is null with a structural redirection: the magnitude residual is not a one-parameter coupling-strength quantity, and the search must address the mode-selection rule rather than a centrifugal coefficient.

## 1 The Probe

**Proposition 1.1** (The two A313 endpoints lie in disjoint  $\mu$ -ranges). *Let  $\hat{O}_{l=n}(g) = -d^2/dr^2 - (3/r)d/dr + gn(n+2)/r^2 + V_{\text{eff}}(r)$ , self-adjoint in  $r^3 dr$ , with the centrifugal term scaled by the scalar  $g \in [0, 1]$  and the operator otherwise verbatim from A313. Two mode-consistent single-parameter families hold the construction fixed while turning  $g$ : family A takes family  $n$  to be the  $n$ -th radial mode of  $\hat{O}_{l=n}(g)$ , so that at  $g = 0$  the three sector operators coincide and the modes are 0, 1, 2 of the pure radial operator, reproducing A313's four-dimensional radial endpoint with  $\mu = 1.222$ ; family B takes family  $n$  to be the ground radial mode of  $\hat{O}_{l=n}(g)$ , so that at  $g = 1$  it is the full  $l = n$  identity, reproducing A313's angular endpoint with  $\mu = 2.009$ . Computed on the*

standard grid, family A runs  $\mu$  monotonically from 1.222 at  $g = 0$  to 0.975 at  $g = 1$ , spanning  $[0.975, 1.222]$ ; family B runs  $\mu$  monotonically from 2.532 at  $g = 0$ , where its three families collapse to a common degenerate ground mode, to 2.009 at  $g = 1$ , spanning  $[2.009, 2.532]$ . The two ranges are disjoint and the target  $\mu = 1.5$  lies in the gap between them. Neither family crosses 1.5 for any  $g \in [0, 1]$ , so no crossing  $g^*$  exists. With no  $g^*$  defined, the ten pre-declared corpus constants have nothing to be compared against, and the fit-versus-forced criterion is vacuous: there is neither a fitted coupling nor a forced one.

**Remark 1.2** (The residual is in the mode-selection rule, not a coupling strength). A313 located strong magnitude motion in the angular sector and read the  $1.222 \rightarrow 2.009$  change as the effect of a centrifugal coupling carried past the target. A315 isolates the source of that change. When the centrifugal strength is the only quantity varied, with the mode-selection rule held fixed, the magnitude moves in the opposite direction and stays inside one of two disjoint ranges that exclude the target. The whole of the  $1.222 \rightarrow 2.009$  motion came from the change in which radial mode each family occupies, from modes 0, 1, 2 of a single operator to the ground mode of three distinct  $l$ -sector operators, not from the strength of the centrifugal term. The honest consequence is that the magnitude residual to 1.5 cannot be expressed as a single angular-coupling coefficient, and the question of whether such a coefficient is a corpus constant does not arise. There is no number to overclaim. The closest fit on the full target is unchanged from A313 and A311, the four-dimensional radial operator, and within the families scanned here family A at small  $g$  reduces the fit distance slightly while leaving  $\mu$  below the target and restoring no diagonal peak. The redirection this records is concrete: the next quantity to fix is the mode-selection rule that maps family  $n$  to a radial state, the rule that carries the  $1.222 \rightarrow 2.009$  motion, rather than a centrifugal coefficient interpolating between the two A313 readings.

**Status.** OI-287-1 remains open. A315 closes one proposed route: the magnitude residual to  $\mu = 1.5$  is not a one-parameter angular-coupling strength. The two A313 endpoints differ in mode-selection, not centrifugal strength, and the two mode-consistent single-knob families span disjoint  $\mu$ -ranges  $[0.975, 1.222]$  and  $[2.009, 2.532]$  that straddle the target without reaching it, so no crossing coupling  $g^*$  exists and the fit-versus-forced test on  $g^*$  is vacuous. The magnitude motion lives in the mode-selection rule, not in a centrifugal coefficient. The closest fit is still the pure radial four-dimensional operator of A311, and the diagonal peak remains unrecovered. No published number changes. The remaining quantity is the mode-selection rule for the family-to-state map, together with the unrecovered peak.

**Verifier.** `verify_P315.py`: recomputes both families from scratch and confirms family A reproduces the four-dimensional radial endpoint at  $g = 0$  ( $\mu = 1.222$ ), family B reproduces the  $l = n$  endpoint at  $g = 1$  ( $\mu = 2.009$ ), each family is monotone in  $g$ , family A stays below 1.5 with maximum 1.222 and family B stays above 1.5 with minimum 2.009, the two  $\mu$ -ranges are disjoint so the  $1.222 \rightarrow 2.009$  motion is mode-reindexing, no single-knob family crosses  $\mu = 1.5$  on  $[0, 1]$ , the verdict is null so there is no  $g^*$  to test against the ten pre-declared corpus constants, and no NaN appears. Eight checks, all PASS.