

Addendum 313: The Full Arena

Adding the angular and fiber sectors moves the magnitude past target, not onto it

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case B (strengthened partial; the angular sector carries the residual motion but the forced $l = n$ linkage overshoots, OI-287-1 magnitude characterized, not closed) — **Addresses:** OI-287-1 — **Verifier:** `verify_P313.py` — all PASS

Abstract

A311 forced the inner-product measure of the A299 selection rule to the radial volume element $r^3 dr$ of the B^4 arena and left two limits. The lift stops at $\mu = 1.337$ rather than reaching 1.5, and no measure restores the column-standardized diagonal peak. A311 read the four-dimensional radial operator's improved fit at $L^2 = 0.399$ as a sign that the residual is shared with the angular and Hopf-fiber sectors that the pure radial reduction drops. This addendum tests that reading. Paper 18 S2.2 gives the bulk radial equation with the angular term $[-d^2/dr^2 - (3/r) d/dr + l(l+2)/r^2]R = ER$, and S4.3 the explicit assembled form $\hat{O}_{l,k} = -d^2/dr^2 - (3/r) d/dr + l(l+2)/r^2 + V_{\text{eff}}(r) + \omega^k \gamma$, with $\omega = e^{2\pi i/3}$ the cube root of unity and $\gamma = 3/4$ the layer-cycle coefficient. The radial node index and the angular momentum enter the operator together through the unperturbed spectrum $E_{n,l} = (2n + l + 2)^2$, and the three families are the three layer-cycle eigenspaces of S5.3. The geometrically forced reading of family n in the full arena is therefore $l = n$ in the angular sector and $k = n$ in the fiber, each family the ground radial mode of its own l -sector operator. Five forced candidates run without fitting: the flat control, the A311 r^3 measure, the A311 four-dimensional radial operator, the angular full-arena candidate, and the Hopf-fiber full-arena candidate with the lapse measure $r^3 |\sin \pi r|$ and the $\omega^k \gamma$ family shift. The angular sector carries strong magnitude motion, confirming the direction of A311's reading, but the $l = n$ assignment moves the magnitude past the target rather than onto it. The angular candidate returns $\mu = 2.009$, the fiber candidate $\mu = 2.265$, both with τ inflating to 18.6 and 21.2 and the fit distance growing to $L^2 = 10.6$ and 13.2. The angular sector supplies most of the motion, a shift of +0.672 in μ against the fiber's +0.256. No candidate restores a valid diagonal peak, so the pre-registered HIT criterion is met by none, and the closest fit remains the pure radial four-dimensional operator at $L^2 = 0.399$. The verdict is strengthened-partial: the residual to 1.5 is partly in the dropped angular sector, the angular sector supplies most of the motion, and the simple forced $l = n$ linkage over-applies it. The magnitude piece of OI-287-1 is characterized, not closed.

1 The Probe

Proposition 1.1 (The angular sector carries the residual motion and overshoots). *Let $B_{nk}^W = \langle \phi_n | c_k \pi^k r^k | \phi_n \rangle_W / \langle \phi_n | \phi_n \rangle_W / \lambda_n$ be the A299-style diagonal overlap in the measure $W dr$, with ϕ_n the family- n mode. In the pure radial reduction ϕ_n is the n -th mode of one operator; in the full arena ϕ_n is the ground radial mode of the $l = n$ sector operator $\hat{O}_{l=n} = -d^2/dr^2 - (3/r) d/dr + n(n+2)/r^2 + V_{\text{eff}}(r) + \omega^n \gamma$ of P18 S4.3, self-adjoint in $r^3 dr$ for the angular candidate and in the Hopf-lapse measure $r^3 |\sin \pi r| dr$ for the fiber candidate. The flat control $W = 1$ returns the A299*

diagonal (1, 1.12, 7.50) at $L^2 = 0.628$. The forced volume $W = r^3$ returns (1, 1.337, 8.706) with $\mu = 1.337$, and the four-dimensional radial operator in its r^3 measure returns (1, 1.222, 8.286) at $L^2 = 0.399$, the closest of all, reproducing A311. The full-arena angular candidate, family n in the $l = n$ sector, returns (1, 2.009, 18.635) with $\mu = 2.009$ at $L^2 = 10.65$. The full-arena fiber candidate returns (1, 2.265, 21.188) with $\mu = 2.265$ at $L^2 = 13.21$. The angular sector supplies a shift of +0.672 in μ relative to the pure radial r^3 value; the fiber adds a further +0.256. Both full-arena values exceed the target 1.5. No candidate satisfies the column-standardized diagonal-peak criterion paired with the failing sine-box control, so the pre-registered HIT criterion, a valid peak together with all three entries within twenty percent of (1, 1.5, 8), is met by none.

Remark 1.2 (The residual is in the angular sector, the linkage over-applies it). A311 located the lift in the forced arena volume and read the residual gap to 1.5 as carried by the angular and fiber sectors the radial reduction drops. A313 confirms the direction and refines the reading. The angular sector does carry magnitude: moving family n into its own $l = n$ sector shifts μ by +0.672, the largest single move in the arc, and the fiber adds a further +0.256 through the lapse measure and the $\omega^k\gamma$ family term. The residual to 1.5 is therefore not in the measure normalization alone; it is in the sector structure A311 named. The limit is that the forced $l = n$ linkage moves the magnitude past the target rather than onto it. The centrifugal term $l(l+2)/r^2$ pushes the higher- l ground modes outward toward the boundary, where the cubic head of ρ is largest, so the degree-three overlap grows faster than the degree-two one and both grow past the target. The naive identity of the family index with the angular momentum is too strong a coupling; the overlap responds to it more than the target asks. The honest statement is that the magnitude motion lives where A311 said it would, in the dropped sectors, and that the correct strength of the angular coupling is weaker than the forced $l = n$ identity. The closest fit remains the pure radial four-dimensional operator at $L^2 = 0.399$, which neither overshoots nor restores the peak. The diagonal peak is unrecovered by every candidate. The magnitude piece of OI-287-1 is thereby characterized rather than closed: the sector is identified, the strength is not.

Status. OI-287-1 remains open. The A311 reading that the residual sits in the angular and fiber sectors is confirmed in direction: the angular sector supplies most of the magnitude motion, a shift of +0.672 in μ , and the fiber adds +0.256. The forced $l = n$ identity of P18 S4.3 over-applies the coupling, carrying μ to 2.009 for the angular candidate and 2.265 for the fiber, past the target 1.5, with τ and the fit distance degrading. No candidate restores the diagonal peak, and the closest fit is still the pure radial four-dimensional operator of A311 at $L^2 = 0.399$. The remaining quantity is the correct strength of the angular coupling, weaker than the forced $l = n$ identity, together with the unrecovered peak. No published number changes.

Verifier. `verify_P313.py`: recomputes the flat eigenstructure (eigenvalues ordered (16.51, 51.24, 102.02), node counts (0, 1, 2)), confirms the flat control reproduces (1, 1.12, 7.50), confirms the forced r^3 measure gives $\mu = 1.337$ and the four-dimensional radial operator gives $\mu = 1.222$ at $L^2 = 0.399$ as the closest candidate (A311), confirms the full-arena angular candidate moves μ to 2.009 past the target, the fiber candidate to 2.265 further still, and that the angular sector supplies the larger share of the motion (+0.672 against +0.256), and confirms the strengthened-partial conditions, that a full-arena candidate exceeds $\mu = 1.40$ while no candidate yields a valid diagonal peak and the magnitude overshoots rather than closes. Eight checks, all PASS.