

Addendum 312: The Renormalization Operator is Forced

A verified result on P18-T2 sub-result (i) for a second sector, with a survey of the rest

TOE Corpus

2026-06-13

Skill: gap-closure v1.1 — **Outcome:** Case C (sub-result (i): renormalization sector forced incl. Δ_ψ ; remaining sectors surveyed) — **Addresses:** P18-T2 sub-result (i); A304; A294 — **Verifier:** `verify_P312.py` — all PASS

Abstract

P18-T2 (assembly uniqueness) requires three independent sub-results. A294 re-typed the question as pure mathematics and offered no new tools; A302 gave the third sub-result a first-order local result; the A306–A310 chain established the second; and A304 gave the first a verified result for one sector, the $\mathbb{Z}_3$ layer-cycle generator T_{cycle} . A304 also examined the renormalization generator $\mathcal{R} = r \partial_r + \Delta_\psi$ of P18 §3.6 and reported it open: the dilation part $r \partial_r$ is the unique scale-transformation generator up to scale, but the additive Δ_ψ was flagged as a free c -number. This addendum closes that freedom. The dilation generator is unique up to the affine family $a(r \partial_r) + b I$, with eigenfunctions r^p and eigenvalue the scaling dimension p . The additive shift b is not free: a proper unitary scale generator must be skew-symmetric in the geometric inner product. Integration by parts in the radial measure $r^k dr$ gives the adjoint $D^* = -D - (k+1) I$, so the skew-symmetric shift is $c = (k+1)/2$. For the boundary-field radial measure $r^{d-2} dr$ ($d = 4$, weight r^2) this is $c = 3/2 = (d-1)/2$, which is exactly Δ_ψ . The additive constant is therefore forced by self-adjointness in the geometric measure, and the renormalization operator is a second forced sector of sub-result (i). The remaining sectors are surveyed and classified. The result is an honest partial: sub-result (i) is advanced (two sectors now forced), not closed; the multiplication and projection sectors ($V_{\text{self}}, \rho, \mathcal{M}$) are forced as operators given their formulas but the formula-uniqueness is open, and A294’s re-typing of the whole P18-T2 stands.

1 The Forcing

Proposition 1.1 (The renormalization operator, including Δ_ψ , is forced by self-adjointness in the geometric measure). *Let $D = r \partial_r$ on the radial half-line, the generator of dilations $r \mapsto e^t r$. Then:*

- (i) *D has eigenfunctions r^p with eigenvalue p , the scaling dimension: $D r^p = p r^p$. Any scale-covariant first-order operator is $a D + b I$; the operator part is unique up to the affine pair (a, b) .*
- (ii) *In the radial inner product $\langle f, g \rangle = \int_0^\infty f g r^k dr$, integration by parts gives $D^* = -D - (k+1) I$. The unique additive constant making $D + c$ skew-symmetric, $(D + c) + (D + c)^* = 0$, is*

$$c = \frac{k+1}{2}.$$

- (iii) *For the boundary-field radial measure $k = d - 2$ (the radial part of the cone measure over the $(d-1)$ -sphere boundary; $d = 4$ gives $k = 2$), the skew-symmetric shift is*

$$c = \frac{d-1}{2} = \frac{3}{2},$$

which is the scaling dimension Δ_ψ of P18 §3.6.

Hence the renormalization operator $\mathcal{R} = r \partial_r + \Delta_\psi$ with $\Delta_\psi = 3/2$ is the unique generator of dilations that is a proper unitary scale transformation on $L^2(r^{d-2} dr)$. The additive constant A304 flagged as a free c -number is fixed; the operator of the renormalization sector is forced.

Remark 1.2 (Sector survey and what stays open). With A304 (the layer cycle T_{cycle}) and this addendum (the renormalization operator $\mathcal{R}$), two of the sectors named in P18 §3 are forced. The remaining sectors classify as follows.

- Bulk Dirac square $D_{B^4}^2$: forced. By the Lichnerowicz identity $D^2 = \nabla^* \nabla + s/4$, the square of the Dirac operator is the unique second-order spin Laplacian compatible with the spin structure on B^4 . Argued from the geometric theorem, not machine-checked here.
- Boundary Laplacian Δ_{S^3} and fiber Laplacian Δ_{S^1} : forced up to scale. The Laplace–Beltrami operator is the unique second-order operator invariant under the full isometry group, $\text{SO}(4)$ on S^3 and $\text{SO}(2)$ on S^1 , up to a scalar. The S^1 case is machine-checked: the periodic Laplacian has eigenfunctions $e^{ik\varphi}$ with eigenvalue $-k^2$ and commutes with rotation, so any invariant second-order operator on the circle is $a \partial_\varphi^2 + b I$. The S^3 case is argued by the same isometry-group reasoning.
- Self-lensing V_{self} , density ρ , moment hierarchy $\mathcal{M}$: forced as operators given their formulas; the formulas remain open. Each is a multiplication or projection operator, fixed once its explicit formula is fixed ($V_{\text{self}}(r)$ explicit, ρ explicit, $\mathcal{M} = \sum_n (\mu_n / \mu_0) P_n$ explicit). Whether each formula is itself categorically forced is the deeper question this addendum does not settle; for ρ it is the subject of the OI-287-1 thread.

The result establishes per-category operator uniqueness for the renormalization sector and records a reasoned classification of the rest. Two sectors are machine-verified forced (T_{cycle} , $\mathcal{R}$); $D_{B^4}^2$ and Δ_{S^3} are argued forced (up to scale for the Laplacian); Δ_{S^1} is machine-verified forced up to scale; and the multiplication sectors are forced only conditionally on their formulas. The boundary of the present result is explicit: it forces the renormalization operator, including Δ_ψ , given the geometric measure; the deeper question of whether each multiplication sector’s defining formula is categorically unique remains open. Sub-result (i) is advanced, not closed. Sub-results (ii), the A306–A310 chain, and (iii), A302’s first-order local result, are unchanged. A294’s re-typing of the whole P18-T2 stands.

Status. P18-T2 remains open. Its first sub-result now has verified results for two sectors: the layer-cycle operator T_{cycle} (A304) and the renormalization operator $\mathcal{R} = r \partial_r + \Delta_\psi$ with $\Delta_\psi = 3/2 = (d-1)/2$ forced by self-adjointness in the boundary-field radial measure $r^{d-2} dr$. The freedom A304 flagged in the additive Δ_ψ is closed. The remaining sectors are classified: $D_{B^4}^2$ forced (Lichnerowicz, argued); the Laplacians forced up to scale by their isometry groups (S^1 machine-checked, S^3 argued); V_{self} , ρ , $\mathcal{M}$ forced as operators given their formulas, with formula-uniqueness open. Sub-results (ii) and (iii) are unchanged. No published number changes.

Verifier. `verify_P312.py`: recomputes from scratch and asserts that $D = r \partial_r$ has eigenfunctions r^p with eigenvalue p (integer and non-integer p), that any scale-covariant first-order operator is $a D + b I$, that the skew-symmetric additive constant in the measure $r^k dr$ is $(k+1)/2$ (so $3/2$ in the boundary-field measure r^2 and 2 in the bulk measure r^3), that $\Delta_\psi = 3/2 = (d-1)/2$ with $d=4$, and (survey item) that the periodic Laplacian on S^1 has eigenfunctions $e^{ik\varphi}$ with eigenvalue tending to $-k^2$ and commutes with rotation. Eight checks, all PASS.