

Addendum 311: The Geometric Measure

The inner product is the forced volume of the arena, not a searched weight

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (strengthened partial; the measure is forced by the arena, OI-287-1 lead grounded, not closed) — **Addresses:** OI-287-1 — **Verifier:** verify_P311.py — all PASS

Abstract

A307 located the factor missing from the A299 base diagonal (1,1.12,7.50) in the inner-product measure used to form the overlap, lifting μ from 1.12 to 1.34 with the density weight ρdx . A309 found that this weight is not fixed by fit: the closest member of a declared density-type family was the non-canonical power $x dx$, and ρdx sat farther from the target. The question left open was therefore whether any measure is forced by the geometry rather than found by search. This addendum tests the natural candidate. Paper 18 S2 places the master operator on sections over the four-ball B^4 with boundary S^3 , Hopf-fibered, and gives the base metric $ds^2 = dr^2 + r^2 d\Omega_3^2$. The four-volume element is then $r^3 dr d\Omega_3$, and the radial reduction of a rotationally symmetric problem carries the measure $r^3 dr$, the surface of S^3 at radius r scaling as r^3 . This is the geometrically forced weight, fixed by the arena and not chosen. Paper 18 S2.2 gives the matching radial operator $-d^2/dr^2 - (3/r) d/dr$, self-adjoint in $r^3 dr$ in the Sturm-Liouville form $-r^{-3} d/dr(r^3 d/dr)$. Five candidates run without fitting: two forced by the arena, the volume weight r^3 on the canonical operator and the genuine four-dimensional radial operator in its r^3 measure, and three for comparison, the flat control, A307's density ρ , and an S^1 Hopf-lapse weight $r^3 \sqrt{1-r^2}$. The forced volume r^3 lifts μ to 1.337, into the band [1.3, 1.7], by essentially the same amount as A307's density at 1.340. The coincidence is analytic. The density is $\rho(r) = 16\pi^3 r^3 + 3\pi^2 r^2 + 2\pi r$, with leading term $16\pi^3 r^3$ carrying 0.933 of $\rho(1)$, so on $[0, 1]$ the density measure is the arena volume r^3 times a constant to leading order, and A307's lift is the geometric lift in disguise. The four-dimensional radial operator in its own measure gives the closest fit of all candidates at $L^2 = 0.399$. No measure restores a valid diagonal peak, so the pre-registered HIT criterion is met by none. The verdict is strengthened-partial: the measure that carries the lift is the forced volume of the B^4 arena, established as geometry rather than fit, and the magnitude piece of OI-287-1 reduces to using the right arena measure, short of the diagonal-peak recovery that would close it.

1 The Probe

Proposition 1.1 (The forced arena volume r^3 carries the A307 lift). *Let $B_{nk}^W = \langle \psi_n^W | c_k \pi^k x^k | \psi_n^W \rangle_W / \langle \psi_n^W | \psi_n^W \rangle_W / \lambda_n^W$ be the A299-style diagonal overlap formed in the measure $W dx$, with ψ_n^W and λ_n^W drawn from the generalized eigenproblem $H\psi = \lambda W\psi$ for the canonical operator $H = -d^2/dx^2 + \rho'(x)^2/(2\Omega^2) + E_{\text{self}} x^2(1-x)^2$. The four-volume element of the base B^4 with metric $ds^2 = dr^2 + r^2 d\Omega_3^2$ fixes the radial measure $W_{\text{geom}} = r^3$, not the flat dx . Evaluated on the five declared weights, the flat control $W = 1$ returns the A299 diagonal (1, 1.12, 7.50) at $L^2 = 0.628$. The forced volume $W = r^3$ returns (1, 1.337, 8.706) with $\mu = 1.337$ in the band [1.3, 1.7]. The genuine four-dimensional radial*

operator $-d^2/dr^2 - (3/r)d/dr + V$, self-adjoint in $r^3 dr$ and solved in its own measure, returns (1, 1.222, 8.286) at $L^2 = 0.399$, the smallest distance of any candidate. The comparison density measure $W = \rho$ returns $\mu = 1.340$, reproducing A307. The Hopf-lapse weight $W = r^3\sqrt{1-r^2}$ returns $\mu = 1.179$, below the band. The forced volume and the density agree to within 0.003 in μ because $\rho(r) = 16\pi^3 r^3 + 3\pi^2 r^2 + 2\pi r$ has leading term $16\pi^3 r^3$, which carries 0.933 of $\rho(1)$; the density measure is the arena volume to leading order. No candidate satisfies the column-standardized diagonal-peak criterion paired with the failing sine-box control, so the pre-registered HIT criterion, a valid peak together with all three entries within twenty percent of (1, 1.5, 8), is met by none.

Remark 1.2 (The measure is forced, not fitted). A309 left the measure underdetermined by fit, the closest weight being the non-canonical power $x dx$. A311 changes the question from which weight fits to which weight the arena requires. The four-ball B^4 with its round- S^3 slices has one radial volume element, $r^3 dr$, and it is not a parameter. That this forced weight lifts μ to 1.337, the same lift A307 read from the density ρ , is the substantive result: the two are the same measure at leading order, and the density coupling of P18 S3.5 carries the geometry of the bulk through its cubic head. The flat dx of A299, by contrast, is the one un-geometric choice, the radial measure of a line rather than of a four-ball, and it is exactly the choice that leaves μ at 1.12. The reading of A307 is thereby grounded. The factor lives in the measure, and the measure is the volume of the arena. Two limits remain. The lift stops at 1.34 rather than reaching 1.5, and no measure in the set restores the diagonal peak, so the column structure that distinguishes the operator's eigenfunctions from a generic basis is not yet recovered. The four-dimensional radial operator improves the fit to $L^2 = 0.399$ by sharpening τ toward 8 while holding μ near 1.22, which suggests the residual gap is shared between the measure and the operator's angular and fiber sectors not carried by the radial reduction alone. The honest statement is that the measure is now forced and the magnitude is not yet complete. The geometry fixes the weight; the remaining lift and the peak are the open quantities.

Status. OI-287-1 remains open, with its A307 lead grounded in geometry rather than left as a fit. The inner-product measure that carries the lift is the four-volume element $r^3 dr$ of the B^4 arena, fixed by the base metric $ds^2 = dr^2 + r^2 d\Omega_3^2$ of P18 S2.1 and matched to the radial operator $-d^2/dr^2 - (3/r)d/dr$ of P18 S2.2. The forced volume lifts μ to 1.337, into the partial band, by the same amount as A307's density ρ , which equals the arena volume at leading order since $16\pi^3 r^3$ carries 0.933 of $\rho(1)$. The four-dimensional radial operator in its own measure gives the closest fit at $L^2 = 0.399$. The remaining quantities are the lift from 1.34 to 1.5 and the recovery of the diagonal peak, neither supplied here. No published number changes.

Verifier. `verify_P311.py`: recomputes the flat eigenstructure (eigenvalues ordered (16.51, 51.24, 102.02), node counts (0, 1, 2)), confirms the flat control diagonal reproduces (1, 1.12, 7.50), confirms the forced four-volume weight $W = r^3$ lifts μ to 1.337 inside the band [1.3, 1.7], confirms the four-dimensional radial operator in its r^3 measure moves μ to 1.222 toward 1.5 and gives the closest fit at $L^2 = 0.399$, confirms the analytic fact that ρ has leading term $16\pi^3 r^3$ carrying 0.933 of $\rho(1)$ so r^3 and ρ share the leading behaviour, confirms the comparison density measure reproduces A307's $\mu = 1.34$ and that $W = r^3$ matches it within 0.01, and confirms the strengthened-partial conditions, that a forced arena measure lifts μ into the band while no candidate yields a valid diagonal peak. Eight checks, all PASS.