

Addendum 310: The Linear-Coupling Hinge Discharged to the Strength Definition

A verified result on P18-T2 sub-result (ii): both hinges now grounded in P18

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (sub-result (ii): linear-coupling hinge discharged to the dimensional-strength definition; both hinges now grounded) — **Addresses:** P18-T2 sub-result (ii); A308; A306; A294 — **Verifier:** `verify_P310.py` — all PASS

Abstract

A308 advanced P18-T2's second sub-result, that additive composition is the only admissible way to compose the named sectors, by discharging A306's decoupling axiom to the no-category-bleed coverage requirement IC2: within the linear-coupling class, no category bleed is equivalent to additivity. A308 left one hinge in place. The reduction held inside the linear-coupling class, in which each coupling c_j scales its sector linearly as $c_j S_j$ rather than entering through a nonlinear $f_j(c_j) S_j$ or through S_j 's internal structure. That linear form was retained, not derived. This addendum tests whether the linear form is itself forced by P18's treatment of the couplings. P18 §4.2 fixes $\alpha, \gamma, \zeta, \beta$ as coupling *strengths*, each pinned by geometric and dimensional consistency, and §4.1 writes the density sector as $\alpha \rho(r)$, a strength multiplying a fixed sector operator. A strength multiplying a fixed operator is a degree-1 homogeneous linear prefactor by definition, and that definition has an operator content. Two readings carry it. First, dimensional homogeneity: a strength's per-sector operator response is constant in the coupling, $\partial \hat{O}_j / \partial c_j = S_j$, so the operator second derivative $\partial^2 \hat{O}_j / \partial c_j^2$ is the zero operator; only a degree-1 prefactor has vanishing operator curvature, and a units rescaling $c_j \rightarrow s c_j$ preserves the spectrum structure only for that form. Second, superposition within a sector: a strength satisfies $f_j(2c) = 2f_j(c)$, degree-1 homogeneity. Over a declared family of coupling forms, the linear c , the quadratic c^2 , the $\sqrt{c}$, and the $e^c - 1$, applied to the density sector $\rho(r)$ (R5) over the fixed kinetic-plus-self-lensing operator K at canonical $\alpha = 1/137.036$, only the linear form satisfies both readings: its operator second derivative is 8.9×10^{-4} , the zero operator to the finite-difference floor, and its homogeneity residual is 0, while the quadratic carries operator curvature 17.18 matching $\|2S\|_F$, the $\sqrt{c}$ carries 3.47×10^3 , and the $e^c - 1$ carries 8.66, each with a nonzero homogeneity residual. The linear form is the unique member of the family on both criteria. The renormalization sector (R7) under linear coupling shows the same vanishing curvature, confirming the property is sector independent. Combined with A308, additive composition is forced with both hinges grounded in P18's stated requirements: the R1–R8 independent categories give IC2, and the dimensional-strength definition of the couplings gives the linear form. Three boundaries remain. The premise that the couplings are strengths is P18 §4.2's definition made precise, not a result derived from deeper principles. The family of coupling forms is finite and declared, not every nonlinear form. Sub-results (i) and (iii) are unchanged, with A304's verified result for one sector, A302's local result for the coefficients, and A294's re-typing standing.

1 The Forcing

Proposition 1.1 (The linear coupling form is forced by the dimensional-strength definition). *Write the per-sector operator $\hat{O}_j(c) = K + f_j(c) S_j$ over the fixed kinetic-plus-self-lensing part K and the bare density sector $S = \rho(r)$ (R5), normalized, at canonical $\alpha = 1/137.036$, with the renormalization generator $S_2 = r\partial_r$ (R7) at $\zeta = \alpha^{5/4}$ cross-checked. P18 §4.2 treats the coupling as a strength: a fixed prefactor multiplying a fixed sector operator, pinned by dimensional and geometric consistency. Read this at the operator level two ways.*

- (L1) *Dimensional homogeneity. A strength's per-sector operator response is constant in the coupling, $\partial\hat{O}_j/\partial c = S$, so the operator second derivative*

$$\left\| \frac{\partial^2 \hat{O}_j}{\partial c^2} \right\|_F$$

is the zero operator. Under a units rescaling $c \rightarrow sc$, only the degree-1 prefactor preserves this constancy; a nonlinear f_j acquires c -dependence in the response and breaks the dimensional reading. Across the declared family $\{c, c^2, \sqrt{c}, e^c - 1\}$ the linear form has $\|\partial^2 \hat{O}/\partial c^2\|_F = 8.9 \times 10^{-4}$, the zero operator to the finite-difference floor, with linear $\partial\hat{O}/\partial c$ matching the constant operator $\|S\|_F = 8.59$. Every nonlinear form carries genuine operator curvature: the quadratic $\|\partial^2 \hat{O}/\partial c^2\|_F = 17.18$, matching the analytic $\|2S\|_F$; the $\sqrt{c}$ form 3.47×10^3 , matching $\frac{1}{4}c^{-3/2}\|S\|_F$ to the finite-difference residual; the $e^c - 1$ form 8.66, matching $e^c\|S\|_F$.

- (L2) *Superposition within a sector. A strength means doubling the coupling doubles the sector's contribution, $f_j(2c) = 2f_j(c)$, degree-1 homogeneity. Only the linear form satisfies it: its residual $\max_c |f(2c) - 2f(c)|/|2f(c)|$ is 0, while the quadratic gives 1, the $\sqrt{c}$ gives 0.29, and the $e^c - 1$ gives 1.9×10^{-2} .*
- (L3) *The linear form is the unique member of the family that is both curvature-free (L1) and degree-1 homogeneous (L2). The renormalization sector under linear coupling has the same vanishing curvature, 6.2×10^{-4} , so the property is sector independent. A coupling strength, as P18 §4.2 defines it, is therefore the linear form.*

Within “coupling is a dimensional strength” the linear form is forced.

Remark 1.2 (The assembled chain, and what remains open). *The advance relative to A308 is in the assumption, not the numbers. A308 forced additivity from IC2 within the linear-coupling class and retained the linear form. Here the linear form is replaced by P18 §4.2's own definition of the couplings as dimensional strengths, made precise at the operator level: a strength's per-sector response is constant (L1) and doubling the coupling doubles the sector (L2), and only the linear form meets both. The chain assembles. The R1–R8 statement that the named categories are independent gives IC2, the no-category-bleed condition, which A308 showed is equivalent to additivity within the linear class. The §4.2 statement that the couplings are dimensional strengths gives the linear form, which this addendum shows is forced within the declared family. So additive composition with $c_j S_j$ is forced given (R1–R8 independent categories $\Rightarrow$ IC2) and (couplings are dimensional strengths $\Rightarrow$ linear), with no remaining free assumption beyond the R1–R8 coverage requirements and the definition of a coupling as a strength. Both hinges of sub-result (ii) are now grounded in P18's stated requirements rather than asserted. Three boundaries bound the claim. The premise that the couplings are strengths is P18 §4.2's definition, made precise here, not derived from something deeper that the couplings must be strengths; a treatment in which a coupling enters nonlinearly is outside the definition, not contradicted by it. The family of coupling forms is finite and declared, four*

forms spanning the natural alternatives, not every conceivable nonlinear f_j . The result addresses sub-result (ii) alone: the per-category operator uniqueness (i) carries A304's verified result for one sector and the coefficient uniqueness (iii) carries A302's local result, both unchanged. A294's re-typing of the whole P18-T2 stands.

Status. P18-T2 remains open. Its second sub-result now rests on two hinges, both grounded in P18's stated requirements rather than assumed ad hoc: additive composition is forced by the no-category-bleed reading of R1–R8 (IC2, A308) together with the linear form of the couplings, which follows from P18 §4.2's definition of $\alpha, \gamma, \zeta, \beta$ as dimensional strengths. The forcing of the linear form is exact within the declared family, where the linear form is the unique curvature-free, degree-1 homogeneous member. The premise that the couplings are strengths is P18's definition rather than a derived result, the family of forms is finite, and sub-results (i) and (iii) are unchanged. No published number changes.

Verifier. `verify_P310.py`: rebuilds the density sector of P18's radial operator and the four-member family of coupling forms from scratch, asserts the linear operator second derivative vanishes ($< 10^{-2}$, the finite-difference floor set by the radial stencil) while the quadratic and $e^c - 1$ forms match their analytic curvature to 10^{-4} relative and the $\sqrt{c}$ form to 10^{-2} relative, asserts that only the linear form is degree-1 homogeneous, asserts that the linear form is the unique member of the family on both criteria, and confirms the same vanishing curvature on the renormalization sector. Eight checks, all PASS.