

Addendum 309: The Measure Family

A declared sweep of density-type weights leaves the lift located but the weight unforced

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case D (null-ish; the declared family does not improve on the A307 lead, and no canonical measure is distinguished by fit) — **Addresses:** OI-287-1 — **Verifier:** verify_P309.py — all PASS

Abstract

A307 located the factor missing from the A299 base diagonal (1, 1.12, 7.50) in the inner-product measure used to form the overlap. The density weight ρdx , with the eigenstates drawn from the generalized eigenproblem $H\psi = \lambda\rho\psi$ and the overlap formed in that measure, lifts μ from 1.12 to 1.34, while a change to the eigenstates alone with the overlap held flat does not move μ . A307 left open the exact weight and the recovery of the diagonal peak. This addendum declares a principled family of density-type weights and sweeps it without fitting. The family has nine members: the flat control $W = 1$; the power weights $W = x^a$ for $a \in \{1, 2, 3\}$; the density $W = \rho$ and its powers $W = \rho^{1/2}$ and $W = \rho^2$; and the moment measures $W = x\rho$ and $W = x^2\rho$, the natural weights of P18's moment hierarchy $\mu_n = \int x^n \rho$. Every member is strictly positive on the open interval and so a valid Sturm-Liouville weight; each generalized eigenproblem is solved by the Cholesky reduction $C = L^{-1}HL^{-T}$ with $W = LL^T$. No member restores a valid diagonal peak under the column-standardized criterion paired with the sine-box control, so the pre-registered HIT criterion, a valid peak together with all three entries within twenty percent of (1, 1.5, 8), is met by none. The closest member by L^2 is the linear power $W = x$ at 0.3252, the same value A307 returned and not an improvement; every corpus-canonical density-type member sits farther, ρdx itself at 0.7406. The density measure is therefore not forced by the fit. One structural fact survives the null reading. Over the rho-power sub-family the lift μ peaks at the unit power, $\mu = 1.31$ at $W = \rho^{1/2}$, $\mu = 1.34$ at $W = \rho$, and $\mu = 1.28$ at $W = \rho^2$. The density is a local maximum of the lift, not a monotone crossing of the target. The verdict is null-ish: the declared family does not improve on A307, and the exact weight remains the open quantity.

1 The Sweep

Proposition 1.1 (No declared density-type weight restores the peak, and the canonical density is not the fit minimum). *Let $B_{nk}^W = \langle \psi_n^W | c_k \pi^k x^k | \psi_n^W \rangle_W / \langle \psi_n^W | \psi_n^W \rangle_W / \lambda_n^W$ be the A299-style diagonal overlap formed in the measure $W dx$, with ψ_n^W and λ_n^W drawn from the generalized eigenproblem $H\psi = \lambda W\psi$ for the operator $H = -d^2/dx^2 + \rho'(x)^2/(2\Omega^2) + E_{\text{self}} x^2(1-x)^2$. Evaluated on the nine declared weights, the flat control $W = 1$ returns the A299 diagonal (1, 1.12, 7.50) at $L^2 = 0.6281$. The power weights return $\mu = 1.28, 1.33, 1.34$ for $a = 1, 2, 3$, with $W = x$ closest at $L^2 = 0.3252$. The density and its powers return $\mu = 1.31, 1.34, 1.28$ for $W = \rho^{1/2}, \rho, \rho^2$, the density itself at $L^2 = 0.7406$. The moment weights $W = x\rho$ and $W = x^2\rho$ return $\mu = 1.32$ and $\mu = 1.31$ at $L^2 = 0.75$ and $L^2 = 3.89$. No member satisfies the column-standardized diagonal-peak criterion*

paired with the failing sine-box control, so the HIT criterion is met by none. The closest member by L^2 is the linear power $W = x$, which is not a corpus-canonical density object, and its distance 0.3252 equals the A307 value and does not improve it. Every canonical density-type member sits farther from the target than this non-canonical power. The density measure ρdx is therefore not forced by the fit over the declared family.

Remark 1.2 (The density is a stationary point of the lift, not its minimum). *The sweep separates two readings of "distinguished." The fit reading asks which weight is closest to the target by L^2 , and answers with the non-canonical power $W = x$, leaving the canonical density unforced. The lift reading asks how μ depends on the weight within a one-parameter canonical sub-family, and answers differently. Over the rho-power family $W = \rho^p$ the lift is $\mu = 1.31, 1.34, 1.28$ at $p = \frac{1}{2}, 1, 2$. The value is not monotone in the exponent; it rises to the unit power and falls past it. The density ρ , the corpus-named member, sits at the local maximum of the lift over its own power family. This is a weaker statement than the pre-registered strengthened-partial condition, which required either that the fit minimum be canonical or that μ cross the target monotonically, and neither holds. It is recorded because it is the one place where ρ stands apart from its neighbours by an intrinsic property of the lift rather than by proximity to the target. The honest summary is that A307's locus stands on its own argument, the contrast between the density measure and the Sturm-Liouville eigenstate change, and not on ρ being the closest weight. The exact weight that would carry μ the full way to 1.5 and restore the diagonal peak is not in this family. Neither the power weights, which approach the target in μ but never peak, nor the canonical density weights, which lift μ comparably but fit worse, recover the column structure. The open quantity is unchanged from A307 and the search is not narrowed by this set.*

Status. OI-287-1 remains open. The A307 reading is unchanged: the missing factor lives in the measure used to form the overlap, established there by the contrast between the density measure and the eigenstate-only change, not by any fit. A309 adds that the declared density-type family does not improve on that lead. No member restores the diagonal peak, and the closest member by L^2 is the non-canonical power $W = x$ at 0.3252, the A307 value, so the density ρdx is not forced as the weight. The single structural fact recorded is that μ over the rho-power sub-family peaks at the unit power, so ρ is a local maximum of the lift but not the fit minimum. No published number changes.

Verifier. `verify_P309.py`: recomputes the flat eigenstructure (eigenvalues ordered (16.51, 51.24, 102.02), node counts (0, 1, 2)), confirms the flat control diagonal reproduces (1, 1.12, 7.50), confirms the density measure $W = \rho$ lifts μ to 1.34 reproducing the A307 lead, confirms that no declared measure yields a valid diagonal peak and the HIT criterion is met by none, confirms that the closest non-control member is the non-canonical power $W = x$ at $L^2 = 0.3252$ which does not improve the A307 value so ρdx is not forced by fit, and confirms that the rho-power lift is not monotone in the exponent but peaks at the unit power. Eight checks, all PASS.