

Addendum 308: The Decoupling Axiom Discharged to Coverage

A verified result on P18-T2 sub-result (ii): no category bleed forces additivity

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Skill: gap-closure v1.1 — **Outcome:** Case C (sub-result (ii) strengthened: decoupling axiom discharged to the IC2 coverage requirement) — **Addresses:** P18-T2 sub-result (ii); A306; A294 — **Verifier:** verify_P308.py — all PASS

Abstract

A306 advanced P18-T2’s second sub-result, that additive composition is the only admissible way to compose the named sectors, conditional on one stated assumption: the independent-decoupling axiom, under which each $c_j \rightarrow 0$ removes sector j and leaves the other sectors’ action unchanged. That axiom was assumed. This addendum tests whether it follows from P18’s design requirements (R1)–(R8) of §1.2, which P18 states are categorical coverage requirements, each naming an independent category of structure, not algebraic constraints. The operator reading of “independent category” is no category bleed: the contribution of sector j to the operator is a function of c_j and S_j alone, not of the other couplings c_i (IC2), together with coverage surviving removal, that setting $c_i \rightarrow 0$ does not distort the remaining categories (IC1). The operator-level measure of category bleed between two sectors is the mixed second partial of the operator itself, $B_{ij} = \|\partial^2 \hat{O} / \partial c_i \partial c_j\|_F$, the same operator quantity A306 used as its discriminator. Over a declared, finite, linear-coupling family of four compositions on two sectors of P18’s radial operator, the density coupling $\alpha \rho(r)$ (R5) and the renormalization generator $\zeta r \partial_r$ (R7) at the canonical $\alpha = 1/137.036$, $\zeta = \alpha^{5/4}$, no bleed holds if and only if the composition is additive: the additive B_{ij} is the zero operator to machine precision, the multiplicative B_{ij} is 3.06×10^6 , and the affine-with-product and operadic forms are each 5.47, matching their analytic cross-terms. Within the linear-coupling class IC2 is therefore equivalent to additivity, so the no-category-bleed reading of (R1)–(R8) forces additive composition without a separate decoupling axiom. The advance is strictly stronger than A306 because the assumption is now grounded in P18’s coverage requirements rather than asserted. Three boundaries bound it: couplings are still assumed to act as linear strengths, which is P18’s own treatment; the family tested is finite and declared, not every conceivable operad; and sub-results (i) and (iii) are unchanged with A294’s re-typing standing.

1 The Forcing

Proposition 1.1 (No category bleed is equivalent to additivity, within the linear-coupling class). *Write P18’s radial operator over the fixed kinetic-plus-self-lensing part K and the bare sector operators S_1, S_2 carrying linear coupling strengths c_1, c_2 , restricting to two sectors for a clean demonstration: the density coupling $S_1 = \rho(r)$ (R5) with weight $c_1 = \alpha$ and the renormalization generator $S_2 = r \partial_r$ (R7) with weight $c_2 = \zeta$, at the canonical $\alpha = 1/137.036$, $\zeta = \alpha^{5/4}$. Define the operator-level category-bleed measure*

$$B_{12}(\hat{O}) = \left\| \frac{\partial^2 \hat{O}}{\partial c_1 \partial c_2} \right\|_F.$$

The no-category-bleed condition IC2, the operator reading of “R1 and R7 name independent categories”, is $B_{12} = 0$. The coverage-on-removal condition IC1 is that $\hat{O}(c_1, 0)$ leaves category 1 equal to the bare reduced operator $K + c_1 S_1$. Take the declared linear-coupling family in which each c_j scales its sector linearly, per P18’s treatment of $(\alpha, \gamma, \zeta, \beta)$ as multiplicative strengths:

$$\begin{aligned}\hat{O}_{\text{add}} &= K + c_1 S_1 + c_2 S_2, \\ \hat{O}_{\text{mult}} &= K (I + c_1 S_1)(I + c_2 S_2), \\ \hat{O}_{\text{affp}} &= K + c_1 S_1 + c_2 S_2 + c_1 c_2 S_1 S_2, \\ \hat{O}_{\text{oper}} &= K + c_1 S_1 + c_2 S_2 + \tfrac{1}{2} c_1 c_2 (S_1 S_2 + S_2 S_1).\end{aligned}$$

Then no category bleed holds if and only if the composition is additive:

- (i) The additive composition is affine in (c_1, c_2) , so $B_{12}(\hat{O}_{\text{add}})$ is the zero operator identically, 0 to machine precision; and IC1 holds, the $c_2 \rightarrow 0$ limit recovers $K + \alpha S_1$ with spectral residual 0.
- (ii) Every non-additive member of the family carries a nonzero $c_1 c_2$ cross-structure, so its B_{12} is a forced bleed: $B_{12}(\hat{O}_{\text{mult}}) = 3.06 \times 10^6$, matching the analytic $\|K S_1 S_2\|_F$ to 10^{-6} relative, while $B_{12}(\hat{O}_{\text{affp}}) = B_{12}(\hat{O}_{\text{oper}}) = 5.47$, matching $\|S_1 S_2\|_F$ and $\|\frac{1}{2}(S_1 S_2 + S_2 S_1)\|_F$ to the finite-difference residual.
- (iii) Across the family, $B_{12} = 0$ selects the additive composition alone. The multiplicative form additionally fails IC1, distorting the surviving category on removal (spectral residual 0.72); the affine-product and operadic forms preserve IC1 but are excluded by their nonzero bleed.

Within the linear-coupling class IC2 is therefore equivalent to additivity. The no-category-bleed reading of (R1)–(R8) forces additive composition.

Remark 1.2 (What the result rests on, what remains open). The advance relative to A306 is in the assumption, not the numbers. A306 forced additivity given the independent-decoupling axiom and assumed that axiom. Here the axiom is replaced by IC2, the operator reading of P18 §1.2’s statement that (R1)–(R8) name independent categories of structure. IC2 is not a new postulate; it is what “the categories are independent” says at the operator level, namely that turning on one sector does not enter another sector’s contribution. The forcing is then the family-level equivalence above: no bleed selects the additive form alone. Three boundaries remain. The couplings are assumed to act as linear strengths, which is P18’s own treatment of $(\alpha, \gamma, \zeta, \beta)$ but is an assumption nonetheless; a composition in which a coupling acts nonlinearly is outside the class tested. The family is finite and declared, four compositions chosen to span the natural alternatives; exotic operads engineered so that their cross-terms cancel on the relevant sectors are not enumerated, and the full operadic sweep stays open. The result addresses sub-result (ii) alone: the per-category operator uniqueness (i) carries A304’s verified result for one sector and the coefficient uniqueness (iii) carries A302’s local result, both unchanged. A294’s re-typing of the whole P18-T2 stands. The standing of the question is that sub-result (ii) is now forced by a coverage requirement rather than by a separate axiom, a strictly weaker hypothesis than A306 carried.

Status. P18-T2 remains open. Its second sub-result now rests on a weaker hypothesis than before: additive composition is forced by the no-category-bleed reading of (R1)–(R8), a P18 coverage requirement, rather than by a separately assumed decoupling axiom. The forcing is exact within the declared linear-coupling family, where no bleed is equivalent to additivity. The assumption that couplings act as linear strengths remains, the full operadic sweep remains open, and sub-results (i) and (iii) are unchanged. No published number changes.

Verifier. `verify_P308.py`: rebuilds the two sectors of P18’s radial operator and the four-member linear-coupling family from scratch, asserts the additive operator mixed partial B_{12} vanishes ($< 10^{-4}$) as an exact identity while each non-additive member is a forced bleed ($> 10^{-2}$) matching its analytic cross-term, asserts the $c_2 \rightarrow 0$ coverage check on the additive form, and asserts that within the family no bleed is equivalent to additivity. Eight checks, all PASS.