

Addendum 307: The Measure

A positive localization of the missing factor to the inner product

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (partial localization; OI-287-1 lead refined, not closed) — **Addresses:** OI-287-1 — **Verifier:** `verify_P307.py` — all PASS

Abstract

A305 closed the space of post-overlap reweightings. The factor missing from the A299 base $B_{nk} = \langle \psi_n | c_k \pi^k x^k | \psi_n \rangle / \lambda_n$, diagonal (1, 1.12, 7.50) against the target (1, 1.5, 8), is not a function of the family index alone (A300), not a function of the monomial degree alone (A303), and not a kernel acting on the overlap matrix (A305). The reason A305 gave is structural: a kernel that suppresses off-diagonal entries equals one on the matched pair and so multiplies the diagonal entry by one, leaving it fixed; the lift the target demands raises a diagonal entry, and the localizing information is not present in the overlap matrix to be recovered by any reweighting of it. One locus remains. The factor must enter earlier than the overlap, in the inner product that forms it or in the eigenstates themselves. This addendum tests that locus with six declared candidates and no fitting. The flat measure dx is run as a control and recovers (1, 1.12, 7.50). A density-weighted measure ρdx , with the eigenstates drawn from the corresponding generalized eigenproblem $H\psi = \lambda\rho\psi$ and the overlap and normalization both formed in that measure, lifts μ to 1.34. The power measures $x^2 dx$ and $x^3 dx$ lift μ to 1.33 and 1.34. The linear measure $x dx$ lifts μ to 1.28 and gives the closest overall fit at $L^2 = 0.33$, half the control distance. A change of the eigenstates alone, the Sturm-Liouville weight ρ applied to the operator with the overlap held in the flat measure, leaves μ at 1.11 and does not move it. The dressing of the operator by ρ/Ω moves μ only to 1.14. The signal is therefore located. The lift toward 1.5 comes from the measure in which the overlap is formed, not from a change to the eigenstates left in the flat measure. No candidate produces a valid diagonal peak under the column-standardized criterion paired with the sine-box control, so this is a partial localization and not the closure of OI-287-1. The next probe should search measures rather than eigenstate deformations.

1 The Probe

Proposition 1.1 (The missing lift lives in the measure, not the eigenstates). *Let $B_{nk} = \langle \psi_n | c_k \pi^k x^k | \psi_n \rangle / \lambda_n$ be the A299 base, diagonal (1, 1.12, 7.50), formed in the flat measure dx from the ground states of $H = -d^2/dx^2 + \rho'(x)^2/(2\Omega^2) + E_{\text{self}} x^2(1-x)^2$. For a strictly positive weight $W(x)$ define the W -overlap $B_{nk}^W = \langle \psi_n^W | c_k \pi^k x^k | \psi_n^W \rangle_W / \langle \psi_n^W | \psi_n^W \rangle_W / \lambda_n^W$, with ψ_n^W and λ_n^W drawn from the generalized eigenproblem $H\psi = \lambda W\psi$ and the inner product $\langle f \rangle_W = \int f W dx$. Evaluated on the declared weights, the flat control $W = 1$ returns $\mu = 1.12$; the density weight $W = \rho$ returns $\mu = 1.34$; the power weights $W = x^2$ and $W = x^3$ return $\mu = 1.33$ and $\mu = 1.34$; and $W = x$ returns $\mu = 1.28$ with the smallest distance $L^2 = 0.33$. When instead the eigenstates are changed by the Sturm-Liouville weight $W = \rho$ but the overlap is formed in the flat measure, μ remains at 1.11 and does not move toward 1.5; when the operator is dressed by ρ/Ω with the overlap flat, μ reaches only*

1.14. *The lift therefore tracks the measure in which the overlap is formed and not the change to the eigenstates. No candidate satisfies the column-standardized diagonal-peak criterion paired with the failing sine-box control, so the pre-registered HIT criterion, a valid peak together with all three entries within 20%, is met by none. The result is a partial localization of the missing factor to the inner product.*

Remark 1.2 (Why the measure carries the lift, and what is left). *The contrast between the density measure ρdx and the Sturm-Liouville weight is the decisive reading. Both draw their states from the same generalized eigenproblem $H\psi = \lambda\rho\psi$, so the eigenstates are identical between them. They differ only in the measure used to form the overlap. The measure that carries ρ into the integral lifts μ to 1.34; the one that forms the overlap flat leaves μ at 1.11. The lift is attached to the measure, not to the states. This is consistent with A305 read forward. A305 found that the localizing information is absent from the flat overlap matrix, and a reweighting after the fact cannot supply it. The present result names where it is instead present: in the geometric density inner product, the integral against ρdx , which is the natural pairing of P18's density-coupling sector. The lift is real and points the right way, from 1.12 toward 1.5, but it stops short. The largest single move reaches 1.34, and the closest fit by L^2 comes from the linear measure $x dx$ rather than from ρ itself, which leaves the precise weight underdetermined by this small declared set. No measure in the set restores the diagonal peak, the second half of the criterion, so the column structure that distinguishes the operator's eigenfunctions from a generic basis is not yet recovered. The honest statement is that the locus is now fixed and the magnitude is not. The factor enters in the inner product; the exact weight that carries μ the full way to 1.5 while also restoring the peak is the open quantity.*

Status. OI-287-1 remains open, with its lead refined rather than unchanged. The missing factor is now located in the measure used to form the overlap, not in the eigenstates and not in a post-overlap correction. The density inner product ρdx lifts μ from 1.12 to 1.34 and the power measures lift it comparably, while a change to the eigenstates with the overlap held flat does not move μ . The remaining quantities are the exact weight and the recovery of the diagonal peak, neither supplied by the six declared candidates. No published number changes.

Verifier. `verify_P307.py`: recomputes the flat eigenstructure (eigenvalues ordered (16.51, 51.24, 102.02), node counts (0, 1, 2) for both the flat and the ρ -weighted states), confirms the flat control diagonal reproduces (1, 1.12, 7.50), confirms that the density measure and the x^2 and x^3 measures carry μ into the band [1.3, 1.7] while the Sturm-Liouville eigenstate change with a flat overlap does not move μ , and confirms the partial-signal conditions, that no candidate yields a valid diagonal peak and the closest non-control candidate sits at $L^2 = 0.33$ below the control distance 0.63. All PASS.