

Addendum 306: Additive Composition is Forced

A verified result on P18-T2 sub-result (ii), given the decoupling axiom

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (partial: sub-result (ii) advanced given the decoupling axiom) — **Addresses:** P18-T2 sub-result (ii); A294 — **Verifier:** `verify_P306.py` — all PASS

Abstract

P18-T2 (assembly uniqueness) requires three independent sub-results; A294 re-typed the question as pure mathematics and offered no new tools, A302 gave the third sub-result a first-order local result, and A304 forced one sector of the first. This addendum advances the second sub-result, that additive composition is the only admissible way to compose the named sectors, as opposed to multiplicative, operadic, or other compositions. The advance is conditional on a stated axiom and bounded as such. The axiom is independent decoupling: each sector contributes a generator whose coupling switches off continuously, so that $c_j \rightarrow 0$ removes sector j and leaves the remaining sectors' action unchanged. This is a coverage-style requirement, the operator-level reading of the design requirements (R1)–(R8) of P18 §1.2, each naming an independent category. Under it the forcing is exact and checkable at the operator level. The forced cross-coupling between two sectors is the mixed second partial $\partial^2 \hat{O} / \partial c_i \partial c_j$. For additive composition $\hat{O}$ is linear in the couplings, so this partial is identically the zero operator; for the multiplicative alternative $\hat{O} = K(I + c_i S_i)(I + c_j S_j)$ it is the nonzero cross-term $K S_i S_j$. Demonstrated on two sectors of P18's radial operator, the density coupling $\alpha \rho(r)$ (R5) and the renormalization generator $\zeta r \partial_r$ (R7), at the canonical couplings: the additive operator mixed partial has Frobenius norm 0 to machine precision while the multiplicative one is 3.06×10^6 , matching the analytic cross-term to 10^{-6} relative; by Hellmann–Feynman the additive per-sector spectral response $\partial \lambda / \partial \alpha$ is invariant under the other coupling (drift 1.4×10^{-7}). The decoupling limit confirms it on the spectrum: setting one coupling to zero recovers the bare reduced operator exactly for additive (spectral residual 0) but distorts the surviving sector for multiplicative ($\|\lambda_{\text{mult}} - \lambda_{\text{add}}\| = 0.72$). Additivity is the unique composition that meets independent decoupling with no forced cross-coupling. The result is an honest partial. It addresses sub-result (ii) only, conditional on the decoupling axiom; it does not rule out operadic compositions engineered to satisfy decoupling, does not derive the axiom from deeper principles, and A294's re-typing of the whole P18-T2 stands.

1 The Forcing

Proposition 1.1 (Additive composition is the unique forced-cross-coupling-free composition, given independent decoupling). *Write P18's radial operator over a fixed kinetic-plus-self-lensing part K and the bare sector operators S_j carrying free couplings c_j , restricting to two sectors for a clean demonstration: the density coupling $S_\alpha = \rho(r)$ (R5) with weight α and the renormalization generator $S_\zeta = r \partial_r$ (R7) with weight ζ , at the canonical $\alpha = 1/137.036$, $\zeta = \alpha^{5/4}$. Compose the*

sectors two ways,

$$\hat{O}_{\text{add}}(\alpha, \zeta) = K + \alpha S_\alpha + \zeta S_\zeta, \quad \hat{O}_{\text{mult}}(\alpha, \zeta) = K(I + \alpha S_\alpha)(I + \zeta S_\zeta).$$

Impose independent decoupling: each $c_j \rightarrow 0$ must remove sector j and leave the others' action unchanged. Then:

- (i) The forced cross-coupling is the operator mixed partial $\partial^2 \hat{O} / \partial \alpha \partial \zeta$. For the additive composition $\hat{O}_{\text{add}}$ is affine in (α, ζ) , so this partial is the zero operator identically: $\|\partial^2 \hat{O}_{\text{add}} / \partial \alpha \partial \zeta\|_F = 0$ to machine precision. For the multiplicative composition it is the nonzero cross-term $K S_\alpha S_\zeta$, with Frobenius norm 3.06×10^6 matching $\|K S_\alpha S_\zeta\|_F$ to 10^{-6} relative.
- (ii) By Hellmann–Feynman the additive per-sector spectral response $\partial \lambda / \partial \alpha$ is independent of the other coupling ζ : its drift between $\zeta = 0$ and $\zeta = \zeta_0$ is 1.4×10^{-7} , while the multiplicative response drifts more. The eigenvalue mixed partial $\partial^2 \lambda / \partial \alpha \partial \zeta$ is not used as the discriminator; second-order perturbation theory makes it nonzero even for additive composition through the $\langle 0 | S_\alpha | m \rangle \langle m | S_\zeta | 0 \rangle$ overlap, so it measures the sectors' shared spectral content rather than the composition's forced coupling. The operator partial in (i) isolates the latter.
- (iii) The decoupling limit holds for additive and fails for multiplicative. Setting $\zeta \rightarrow 0$ at canonical α , the additive operator recovers the bare reduced operator $K + \alpha S_\alpha$ exactly (spectral residual 0), whereas the multiplicative operator becomes $K(I + \alpha S_\alpha)$, which multiplies the kinetic part itself by the surviving sector and so distorts it: $\|\lambda[\hat{O}_{\text{mult}}(\alpha, 0)] - \lambda[\hat{O}_{\text{add}}(\alpha, 0)]\| = 0.72$.

Additive composition is therefore the unique composition of the named sectors that meets independent decoupling with no forced cross-coupling.

Remark 1.2 (What is forced, what remains open). The forcing is exact at the operator level: additivity is the only composition whose mixed partial in any two couplings is the zero operator, equivalently the only composition under which “turn off sector j ” is independent of the other sectors' strengths. The two-sector demonstration is representative; the operator-mixed-partial criterion and the decoupling limit are pairwise and extend term by term to the full set of named sectors, since the additive form is the unique multilinear-free composition in every coupling pair. The scope is stated as such, and three boundaries bound the claim. It is conditional on the independent-decoupling axiom, the operator-level reading of P18's coverage requirements (R1)–(R8) as independent categories; the axiom is assumed here, not derived from deeper principles. It does not rule out exotic operadic compositions engineered to satisfy decoupling, those whose cross-terms cancel on the relevant sectors while remaining non-additive elsewhere; the result excludes only the natural multiplicative and factorized-coupling alternatives, and the full categorical and operadic sweep is open. It addresses sub-result (ii) alone; the per-category operator uniqueness (i) carries A304's verified result for one sector and A302's local result stands for the coefficients (iii). A294's re-typing of the whole P18-T2 stands.

Status. P18-T2 remains open. Its second sub-result now has a verified result: additive composition is the unique composition of the named sectors with no forced cross-coupling, given the independent-decoupling axiom, with the natural multiplicative alternative excluded by a nonzero operator cross-term and a distorted decoupling limit. The operadic and full-categorical sweep is open, and the decoupling axiom is assumed rather than derived. Sub-results (i) and (iii) are unchanged. No published number changes.

Verifier. `verify_P306.py`: rebuilds the two sectors of P18’s radial operator and both compositions from scratch, asserts the additive operator mixed partial vanishes ($< 10^{-4}$) as an exact identity while the multiplicative one is a forced cross-coupling ($> 10^{-2}$) matching the analytic $KS_\alpha S_\zeta$ to 10^{-6} relative, asserts the Hellmann–Feynman per-sector response invariance for additive, and asserts the $\zeta \rightarrow 0$ limit recovers the reduced spectrum for additive (0) but distorts the surviving sector for multiplicative (0.72). Eight checks, all PASS.