

Addendum 302: Coefficient Rigidity

A first-order local result on P18-T2 sub-result (iii), and the soft direction

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case C (partial advance: first-order local uniqueness proven for one of three sub-results; soft direction named) — **Addresses:** P18-T2 sub-result (iii); A294 — **Verifier:** `verify_P302.py` — all PASS

Abstract

P18-T2 (assembly uniqueness) requires three independent sub-results; A294 re-typed the question as pure mathematics and offered no new tools. This addendum makes a first-order local advance on the third sub-result, that the four coefficients $(\alpha, \gamma, \zeta, \beta)$ are the only spectrum-matching assignment. Treating the four as free assembly weights on their sectors of P18's radial operator (density ρ , layer-cycle ω^k , renormalization $r\partial_r$, moment μ_n/μ_0), with the fixed sectors held canonical, the spectrum-response Jacobian $J_{ij} = \partial\lambda_i/\partial c_j$ at the canonical point has full rank 4: the smallest normalized singular value is bounded away from zero ($\sigma_{\min}/\sigma_{\max} = 9.3 \times 10^{-3}$ on the $l = 0$ sector), so there is no flat direction and the canonical coefficients are the locally unique spectrum-matching assignment. The result strengthens as angular sectors are added, $9.3 \times 10^{-3} \rightarrow 2.3 \times 10^{-2} \rightarrow 3.5 \times 10^{-2}$ for $l \leq 0, 1, 2$, the spectral image of P18's coverage requirements increasing rigidity. One structural feature persists and is recorded as the soft direction: the density coupling α and the renormalization generator ζ are spectrally collinear to 0.9986, both acting as nearly mode-independent shifts. Local uniqueness is carried by the ground-state mode, where the density response separates from the renormalization response. This is local and first order; it does not establish global uniqueness, and sub-results (i) and (ii) remain exactly as A294 left them.

Erratum (verifier sweep, 2026-06-15): The verifier `verify_P302.py` builds the radial operator on a *Dirichlet wall* ($r \in \text{linspace}(dr, 1-dr, N-1)$), not the corpus APS boundary condition (A329). The full-rank-4 *local* coefficient-rigidity result SURVIVES under APS: A330/A335/A336 redo the Jacobian on the corpus operator and local uniqueness stands. However, the α - ζ collinearity reported here at 0.9986 (and the “soft direction” fragility it implies) is a *Dirichlet artifact*: under the corpus APS operator the collinearity is 0.662 (A336), and the soft-direction fragility is DISSOLVED by the corpus tie $\zeta = \alpha^{5/4}$ (A330). The numbers in the abstract, Proposition, and Remark below are retained as the historical Dirichlet record; for the corpus-faithful (APS) treatment see A330/A335/A336.

1 The Rigidity Test

Proposition 1.1 (Local coefficient uniqueness on the spectrum). *Write P18's radial operator with the four named sectors carrying free weights,*

$$\hat{O}_{l,k}(c) = -\frac{d^2}{dr^2} - \frac{3}{r} \frac{d}{dr} + \frac{l(l+2)}{r^2} + V_{\text{self}}(r) + \alpha \rho(r) + \zeta r \frac{d}{dr} + \gamma \text{Re } \omega^k + \beta \frac{\mu_n}{\mu_0},$$

$\omega = e^{2\pi i/3}$, with the kinetic and self-lensing sectors fixed. Let $\lambda_{n,k}(c)$ be the spectrum and $J = \partial\lambda/\partial c$ the response Jacobian over modes (n, k) and coefficients $c = (\alpha, \gamma, \zeta, \beta)$, evaluated at the canonical $(\alpha, \gamma, \zeta, \beta) = (1/137.036, 3/4, \alpha^{5/4}, 3\pi/20)$. The column-normalized J has four nonzero singular values with $\sigma_{\min}/\sigma_{\max} = 9.3 \times 10^{-3} > 0$ on $l = 0$; the rank is exactly 4. No coefficient combination leaves the spectrum invariant to first order: the canonical assignment is locally unique. Including $l = 1$ and $l = 2$ raises the ratio monotonically to 3.5×10^{-2} .

Remark 1.2 (The soft direction, and the limits of this result). *The four sector responses are not equally distinct. The density coupling and the renormalization generator produce nearly the same spectral signature: their normalized response columns have inner product 0.9986, stable across the added angular sectors. Both act as almost mode-independent shifts of the spectrum, and what separates them, hence what carries local uniqueness, is the ground-state mode alone, where the density response is lower ($\partial\lambda_0/\partial\alpha \approx 117$ against ≈ 136 for excited modes). The honest reading: coefficient uniqueness holds locally but is soft along the density/renormalization plane, and a proof of the third sub-result will have to control exactly that pair. Three caveats bound the claim. It is local, a statement about the Jacobian at the canonical point, not global injectivity. It is first order, silent on second-order coincidences. It addresses only sub-result (iii); the per-category operator uniqueness (i) and the additive-composition uniqueness (ii) are untouched, and A294’s re-typing of the whole question stands.*

Status. P18-T2 remains open. Its third sub-result now has a first-order local result in its favor: the coefficients are locally the unique spectrum-matching assignment, strengthening with sector coverage, soft along the α - ζ plane. Sub-results (i) and (ii) are unchanged. No published number changes.

Verifier. `verify_P302.py`: rebuilds the response Jacobian on the $l = 0, 1, 2$ sectors, asserts full rank 4 with $\sigma_{\min}/\sigma_{\max} > 10^{-3}$, asserts the ratio strengthens with added sectors, and asserts the α - ζ collinearity at 0.99+ as the soft direction. All PASS.