

Addendum 299: The Inverse-Eigenvalue Lead

A sixth excluded class, and the closest approach to (2, 3, 16)

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case D (mechanism class excluded; OI-287-1 lead sharpened to a single combination) — **Addresses:** OI-287-1 — **Verifier:** `verify_P299.py` — all PASS

Abstract

A298 established that derivative and nodal functionals recover the (2, 3, 16) direction the amplitude reversed, but are monotone and do not peak on the diagonal. A296's surviving coupling strength is $\kappa\Omega/\lambda_1$, which names eigenvalue data. This addendum asks whether a fixed eigenvalue weighting of the right-direction coupling localizes it. Two right-direction base matrices (the derivative overlap and the density-Taylor overlap) are each weighted by four declared functions of the eigenvalues λ_n , no fitting. None produces a diagonal peak, so a sixth mechanism class is excluded. The result is the closest approach yet. The combination singled out by the framework itself, the density-Taylor overlap $\langle\psi_n|c_k\pi^k x^k|\psi_n\rangle$ divided by λ_n , which is the level-resolved form of the A296 strength, gives the normalized diagonal (1, 1.12, 7.50) against the target (1, 1.5, 8), an L_2 distance of 0.63, with the τ entry off by six and a quarter percent. Every factor in this combination is named by the corpus; nothing is tuned. The two open gaps are explicit: the μ entry misses by twenty-five percent, and the construction still does not peak on the diagonal. The selection rule's missing mechanism, if it exists in this operator, is a refinement of this one combination.

1 The Probe

Proposition 1.1 (No fixed gap weighting peaks; one nearly hits). *Take the right-direction base matrices of A298: the derivative overlap $D_{nk} = \langle\psi'_n|x^k|\psi'_n\rangle$ and the density-Taylor overlap $R_{nk} = \langle\psi_n|c_k\pi^k x^k|\psi_n\rangle$. Weight each row n by one of four declared functions of λ_n : $1/\lambda_n$, $1/(\lambda_n - \lambda_1)$, λ_n/λ_1 , $1/\sqrt{\lambda_n}$. Across all eight combinations, none attains its column-standardized row maximum on the diagonal: a sixth mechanism class is excluded. The smallest L_2 distance from the normalized diagonal to the target (1, 1.5, 8) is achieved by R weighted by $1/\lambda_n$, the level-resolved form of $\kappa\Omega/\lambda$: its diagonal is (1, 1.12, 7.50), $L_2 = 0.63$, with τ off by six and a quarter percent. No other combination comes within $L_2 = 5.9$.*

Remark 1.2 (Why this combination is privileged, and what is missing). *The winning combination is not one of eight equals. The density-Taylor overlap is the only base that reads ρ 's actual coefficients $c_k\pi^k$ rather than bare monomials, and $1/\lambda_n$ is the only weighting that is the corpus's own coupling strength resolved to the level. That the framework's own pieces, assembled with no freedom, land within six and a quarter percent on the heaviest family is the strongest evidence to date that the selection rule lives in this operator. Two gaps remain and are not excused. The μ*

entry, 1.12 against 1.5, is twenty-five percent low, and the off-diagonal structure does not concentrate on the diagonal. The honest reading is that the $(2, 3, 16)$ magnitudes are nearly present in $\langle \psi_n | c_k \pi^k x^k | \psi_n \rangle / \lambda_n$, and that the remaining work is a localizing factor that lifts the middle entry and sharpens the diagonal, not a search for a different home.

Status. OI-287-1 remains open. Its form after this addendum: *find the factor that lifts the μ entry of $\langle \psi_n | c_k \pi^k x^k | \psi_n \rangle / \lambda_n$ from 1.12 toward 1.5 and concentrates the coupling on the diagonal.* The 8-combination scan is a multiple comparison; the result's weight comes from the winner being the theoretically privileged cell, not from its rank alone. No published number changes.

Verifier. `verify_P299.py`: rebuilds the eight combinations, asserts none peaks on the diagonal, and asserts that the density-overlap-over- λ_n cell is the unique closest to $(1, 1.5, 8)$ at $L_2 < 0.7$ with the others beyond $L_2 = 5$. All PASS.