

Addendum 298: The Nodal Direction

A fifth excluded class, and the recovery of the (2, 3, 16) direction

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case D (mechanism class excluded; OI-287-1 reframed by a recovered direction) — **Addresses:** OI-287-1 — **Verifier:** `verify_P298.py` — all PASS

Abstract

A297 excluded the position-moment reading of the selection rule and established that the charged-lepton families differ by node count, not by amplitude at the center. This addendum tests the door A297 opened: whether a fixed nodal or derivative pairing couples level n to the degree- n monomial. Four pairings are tried with no free knobs: the derivative overlap $\langle \psi'_n | x^k | \psi'_n \rangle$, the kinetic coupling through $-d^2/dx^2$, the density-weighted overlap against the degree- k Taylor term of ρ , and the ladder transition to the ground state. None makes the coupling peak on the diagonal, so a fifth mechanism class is excluded. The result is not empty, however. Where the position moments of A297 gave a diagonal ratio (1, 0.74, 0.59) decreasing with degree, opposite to the target $(c_1, c_2, c_3)/c_1 = (1, 1.5, 8)$, the three derivative and nodal functionals give increasing ratios: (1, 2.82, 5.17), (1, 2.56, 4.78), and, for the density-weighted overlap, (1, 3.48, 46.35). The last reproduces the target's signature jump at degree three. The sign that the amplitude got backwards is recovered the moment the functional reads the nodal structure. The degree information lives where A297 predicted, in the nodes; what remains open is the mechanism that makes the rising coupling peak on the diagonal and land on (2, 3, 16).

1 The Four Pairings

Proposition 1.1 (No diagonal peak from a fixed pairing). *With H , ψ_1, ψ_2, ψ_3 , and the target as in A297, form the 3×3 matrices M_{nk} for: $\langle \psi'_n | x^k | \psi'_n \rangle$; $\langle \psi_n | (-d^2/dx^2) (x^k \psi_n) \rangle$; $\langle \psi_n | c_k \pi^k x^k | \psi_n \rangle$; and $\langle \psi_n | x^k | \psi_1 \rangle$. Standardizing each column across the three levels, none attains its row maximum on the diagonal, and the box control behaves the same. No fixed nodal or derivative pairing reproduces the index-matching arg-max pattern. A fifth mechanism class is excluded, joining A292, A295, A296, and A297.*

2 The Recovered Direction

Proposition 2.1 (Derivative functionals carry the right sign). *The normalized diagonal M_{nn}/M_{11} runs (1, 0.74, 0.59), decreasing, for the A297 position moments, against the target (1, 1.5, 8). For the derivative overlap it runs (1, 2.82, 5.17); for the kinetic coupling (1, 2.56, 4.78); for the density-weighted overlap (1, 3.48, 46.35). All three increase with degree, and the density-weighted overlap reproduces the target's large jump from the second to the third level. The direction of (2, 3, 16) is a property of the nodal and derivative structure, not of the amplitude.*

Remark 2.2 (What the reframing leaves open). *Two facts now bracket OI-287-1. The amplitude has the wrong sign (A297); the nodal and derivative functionals have the right sign but do not peak on the diagonal and do not reach $(2, 3, 16)$ in magnitude (this addendum). The missing element is therefore not the location of the degree information but a localizing factor that makes the rising coupling concentrate on the diagonal. The surviving strength of A296, $\kappa\Omega/\lambda_1$, names eigenvalue data; the natural next test, recorded as the next pre-registration and not performed here, is whether weighting the derivative coupling by the eigenvalue gaps produces the diagonal peak. This is progress of the kind the registry is built to keep: the open problem is strictly smaller than it was, and the direction of its answer is now fixed.*

Status. OI-287-1 remains open. Its form after this addendum: *find the factor that makes the increasing derivative coupling peak on the diagonal at $(2, 3, 16)$.* No published number changes; A287's lepton law and its 1×10^{-4} accuracy stand.

Verifier. `verify_P298.py`: rebuilds the four pairing matrices and their box controls, asserts the absence of a diagonal peak, and asserts that the three derivative and nodal functionals give increasing diagonals while the position moments do not. All PASS.